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Abstract :

This article takes up a large passage from the book Shaft weaving and grah design , by Olivier Masson and François Roussel, first published in 1987 by En Bref. The first part, from chapter 2, and the A of the second part ; from page 31 to 108.

It introduces and develops a mathematical model of weaving, based on relations.


The book Shaft weaving and grah design
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FIRST PART

## MATHEMATICAL MODELS OF THE CLOTH DIAGRAM

## B "PEGPLAN-THREADING-DRAWDOWN" REPRESENTATION

Chapter 2 : Representing the cloth diagram using relations
1- Representation of a diagram by a relation
2- Two restrictive conditions on the relations studied
a) Everywhere defined
b) Surjective

3- Some definitions
a) Mapping
b) Injection
c) Bijection

4- Composition of relations
a) Definition
b) Cloth diagram

5- Reciprocal relation
6- Symmetric relation
7- First diagonal I. Straight diagram. Identical relation I.
8- Reciprocal of the composition of two relations
9- Composite of a relation with its reciprocal
a) Properties
b) "Weaved as drawn in". Threading axial
c) Simplification rules. Invertible relation

10- Involutive relation
11- Second diagonal -I. Return diagram
12- Calculations on straight and return diagrams
13- Geometric transformations of a diagram
a) Expressing a symmetry using -I
b) Expression of any geometric transformation
c) Effect of symmetries on the cloth diagram

Chapter 3: Representing the cloth diagram Using Matrices
1-Definitions
2- Product of matrices
3- Logical operations on relations
a) "Not A" relation
b) Relation "A or B"
c) Relation "A and B"

Chapter 1: Definition
1- Cloth diagram Formula
2- Compatibility of representations
Chapter 2 : Passage from the treadling - tie-up - threading representation to the
peg-plan - threading representation
Multiple cloth diagram

## D FIRST PRACTICAL CONSEQUENCES OF THE TISSUE FORMULA

Chapter 1 : Another Presentation of the cloth diagram
Chapter 2 : Geometric transformations of a diagram
Chapter 3: Warp and weft reversal of a cloth
Chapter 4: Focus on the tie-up
Chapter 5: Generated drawdown
Chapter 6 : Drawdowns symmetrical with respect to the first diagonal
1- "Weaved as drawn in". "Treadling - tie-up - threading - drawdown" representation 2- Condition for a drawdown to be symmetrical with respect to the first diagonal 3-Practical consequences

## SECOND PART

## TRANSFORMATION BASES

## A THEORETICAL STUDY

Chapter 1: Transformations preserving the dimension of the diagram. Amalgamations 1- Bases of rearrangement
2- Amalgamation
3-Multiple amalgamation diagram
4- Equivalent diagrams
Chapter 2: Transformations decreasing the dimension of the diagram. Telescoping
Chapter 3: Transformations increasing the dimension of the diagram.
Diagram combinations

B "PEGPLAN-THREADING-DRAWDOWN" REPRESENTATION

Chapter 2
Representation of the cloth diagram using relations

Several mathematical models of weaving have been developed, based on real numerical functions or parametric equations. If they can shed light on certain aspects of the cloth : its direction of variation, its slope or inclination, the influence of a stretch of threading or treadling, the separation of the cloth into different surfaces, etc. However, two basic criticisms can be made:

- The diagrams are represented by curves. This type of representation is suitable for threadings, but is unable to take into account the full reality of a treadling or a tie-up where the entire surface of the diagram contains information. These models are therefore limited to the description of the graphic curve of a cloth, possibly considered as separating it into several surfaces.
- The curves studied are defined on infinite sets and are continuous at least piecewise. But the reality of weaving is quite different : there is a finite number of threads, shafts, picks or peg-plans. Working on infinite sets of points does not allow us to study a particular point : for example moving a shaft, which greatly affects a cloth, would not make sense in this model.

For these reasons we are going to build a new mathematical model of the cloth, closer to the reality of weaving, in particular a digital model, dealing with finite sets.
This study requires a greater familiarity with mathematics, however its results can be expressed in an intuitive way accessible to all.

## 1- REPRESENTATION OF A DIAGRAM BY A RELATION

a) Representations of diagrams by relations.

In the practice of weaving, the threads, picks, shafts and peg-plans are numbered from 1 ; also we will choose as working sets intervals of N (the set of natural numbers) of the form : [ $1, \mathrm{n}]=$ $(1,2, \ldots, n-1, n)$.


We will consider a diagram A, rectangle of grid paper, of $n$ columns and $p$ lines, as the graph of a relation from $\mathrm{E}=[1, \mathrm{n}]$ in $\mathrm{F}=[1, \mathrm{p}]$.
Columns are numbered from left to right, starting at 1 .
Rows are numbered from bottom to top, starting at 1 .
The domain will therefore be drawn on the width and the codomain on the height. A black square ( $x, y$ ), or a cross in the square ( $x, y$ ), will indicate that $x$ is in relation to $y$ by the relation $A$, we will note x A y (Beware of the meaning ( $\mathrm{x}, \mathrm{y}$ ) is different of ( $\mathrm{y}, \mathrm{x}$ ), we can have x A y and not y Ax ).

Square of coordinate diagram ( $\mathrm{x}, \mathrm{y}$ ) black (contains a cross) <=> x A y

## 2- TWO RESTRICTIVE CONDITIONS ON THE RELATIONS STUDIED

For the sake of simplifying the results and adapting to the practical reality of weaving, we will limit ourselves to the study of diagrams having the following two properties :
a) the diagram will not contain any empty columns.

We will then say that the relation A from E to F is everywhere defined:

$$
\forall x \in E \quad \exists y \in F \quad x A y
$$

b) the diagram will not contain any empty lines

We will then say that the relation A from E to F is surjective :

$$
\forall y \in F \quad \exists x \in E \quad x A y
$$

In weaving, these two conditions are always met :
In a treadling diagram, we don't skip picks, we don't write treadles that are useless.
In a threading diagram all ends are threaded and all shafts are used.
In a tie-up diagram each treadle controls at least one shaft and each shaft is controlled by at least one treadle.

However, during our study, we will encounter diagrams with empty rows or columns. By digitizing a curve or approximating it on a drafting network.


A non-surjective and not everywhere defined relation


A surjective and everywhere defined relation
These diagrams can clearly be reduced to diagrams meeting the above conditions by deleting empty rows or columns. Their properties will be identical (except for holes), however their graphic properties can be hidden.

The relations studied hereafter will therefore be assumed to be everywhere defined and surjective, unless otherwise indicated. Their diagrams will therefore have no empty rows or columns.

## 3- SOME DEFINITIONS

a) We will say of a relation A from E to F that it is a mapping if and only if

$$
\forall x \in E \quad \exists!y \in F \quad x A y
$$



A relation is a mapping if and only if its diagram contains a cross and only one per column.

In weaving, a threading diagram always has this property. Indeed all the ends are threaded in a shaft and only one. A threading diagram is therefore a mapping (surjective, as all relations are assumed here). However, to maintain the generality of our subject, we will also consider the case of any threading diagram, not corresponding to the property of a true threading.
b) We will say of a relation A from E to F that it is injective if and only if

$$
\forall \mathrm{y} \in \mathrm{~F} \quad \forall\left(\mathrm{x}, \mathrm{x}^{\prime}\right) \in \mathrm{E}^{2} \quad \mathrm{xA} \mathrm{~A} \text { and } \mathrm{x}^{\prime} \mathrm{A} y=>\quad \mathrm{x}=\mathrm{x}^{\prime}
$$

An injective relation is therefore a relation which does not include more than one cross per line. As we assume our surjective relations (at least one cross per line), we can state :


> A relation is injective if and only if its diagram contains a cross and only one per line.

Note : we will also speak of injection for an injective relation. Usually the term injection is reserved for mappings, it is clear that we will consider here injective relations which are not mappings (an injection may have several crosses per column).
As we only consider surjective everywhere defined relations, the notions of mapping and injection are perfectly symmetrical : one concerns the columns and the other the rows of the diagram.

A rectangular injection diagram will necessarily have more rows than columns. Indeed, imagine that there are more columns than rows. As the relation is everywhere defined, there is at least one cross per column. There would be more crosses than lines and therefore at least one line containing more than one cross. The relation would therefore not be injective.


Injection
$<=>$ A cross and only one per line
Injection $=>$ width $\leq$ height


Mapping
<=> A cross and only one per column
Mapping $=>$ width $\geq$ height

Similarly, a rectangular mapping, which is assumed to be surjective, will necessarily have more columns than rows.

When do square mappings and injections happen?
c) We will say of a relation A from E to F that it is bijective if and only if it is both a mapping and an injection.

A relation is bijective if and only if its diagram contains a cross and only one per line and per column.


Bijection
<=> A cross and only one per line and per column
Bijection $=>$ width $=$ height
A bijection diagram therefore contains as many crosses as lines and columns; it is square.
Conversely, consider a square diagram, comprising one cross and only one per line, or one cross and only one per column. Our relations are assumed to be everywhere defined and surjective, ie. containing at least one cross per column and per line. Such a diagram will therefore have one cross and only one per line and per column; it is a bijection.

| Square mapping | $<=>$ | Bijection |
| :--- | :--- | :--- |
| Square injection | $<=>$ | Bijection |
| Mapping and injection | $<=>$ | Bijection |

d) Parallel between the language of weaving and that of mathematics.

A diagram is a rectangle of graph paper, it has a width and a height.
Each square can be black or white (checked or unchecked).
Columns are numbered from left to right, starting at 1.
Rows are numbered from bottom to top, starting at 1 .
When it is considered alone we will speak of "weave structure", it is a "unit" of weaving.
It is then considered as a potential drawdown.
Its width is the "repeat in width" of the weave structure and its height its "repeat in height".
A diagram is represented by a relation.
Diagram A square with coordinates ( $\mathrm{x}, \mathrm{y}$ ) black $<=>\mathrm{x}$ A y x is in relation to y by A
When the diagram has a precise function the vocabulary changes.
If it's a "drawdown".
The columns represent the warp threads.
The lines represent the weft threads. A line is also a pick, we pass a weft thread in the shed formed by the warp threads.
A square of graph paper represents an "interlacing", that is to say the intersection of a vertical warp thread and a horizontal weft thread.
A black square is a "riser". The warp threads are lifted, the weft thread going below.
A white square is a "sinker". The warp threads are lowered, the weft thread going above.
The width of the drawdown is equal to the width of the threading.
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Page 9 / 105

The height of the drawdown is equal to the height of the peg-plan.
A warp float is a sequence of consecutive vertical black dots.
A weft float is a sequence of consecutive horizontal white dots.
The term "Cloth" names all the diagrams representing a calculation of cloth ( Peg-plan - Threading Drawdown and possibly tie-up )

If it is a "Threading".
The columns represent the warp threads.
The lines represent the shafts.
A black square indicates that the warp threads of the column are threaded in the shaft (in the heddle carried by the shaft) of the line.
Each end is threaded in a shaft and only one.
Threading is a Mapping
If it's a "Peg-plan".
The columns represent the treadles; or any other device controlling the lifting of the shafts : pegs
dobby, punched cards, electronic dobby.
The lines represent the picks; at each pick we pass a weft thread.
Each treadle, with number $n$, controls the shaft of the line with the same number $n$.
The width of the peg-plan is equal to the height of the threading.
A black square indicates that for this pick, on the line, the treadle of the column raises the shaft of the line of the same number as the column.

If it's a "Tie-up".
The columns represent how the treadles are "tied" to the shafts.
The lines represent the shafts.
The height of the tie-up is equal to the height of the threading.
A black square indicates that the column treadle is attached to the line shaft.
If the treadle is lifted for a pick, the shafts marked in black in that column of the tie-up will be lifted.
If it's a "Treadling". It is then assumed that there is a tie-up.
The treadling is a peg-plan that controls the shafts via the tie-up.
The columns represent treadles that are "tied" to one or more shafts as shown in the tie-up.
The width of the treadling is equal to the width of the tie-up.
The lines represent the picks; with each pick one or more treadles are pushed.
A black square indicates that the shaft is up for a pick, the shafts marked in black in that column of the tie-up will be up.
We may be led to consider a treadling where each pick raises one and only one treadle.
The treadling is then injective.

## 4- COMPOSITION OF RELATIONS

a) Definition

Given a relation A , from E to F :


## B

and a relation B , from F to G :

such that $B$ has for domain, the codomain of $A$;
or again, such that the width of $B$ is equal to the height of $A$,
we can define a relation, from E to G , called composed of A followed by B , denoted B o $\mathrm{A}(\mathrm{read} \mathrm{B}$ round A ), by :

$$
\begin{aligned}
& \text { B o } A=((x, z) \in \operatorname{ExG} / \exists y \in F \quad \text { } \quad \text { A Ay and yBz }) \\
& \forall(x, z) \in \operatorname{ExG} \quad x \text { BoA } z \quad<=>\quad \exists y \in F \quad \text { xAy and yBz }
\end{aligned}
$$



B o A

x z
$B$ o $A$ is a relation from $E$ to $G$; the width of $B$ o $A$ is therefore equal to the width of $A$, the height of B o A is equal to the height of B .

For an element x of E to be in relation by B o A to an element z of G , it is necessary and sufficient that we can find an element $y$ of $F$ which is both in relation with $x$ by A and with z by B . This element is not in general unique. To find all the elements of G which are in relation by BoA with an $x$ of $E$, it suffices to search for all the elements of $F$ in relation with $x$ by $A$, then all the elements of $G$ in relation by $\mathbf{B}$ (With these same elements by F ).

Pay attention to the order of the relations. The element y of F corresponds to a row of A and a column of $B$. The diagonal drawn in the extension of $A$ and $B$ allows us to establish the correspondence between the rows of A and the columns of B . This diagonal is of course square, the width of $B$ being equal to the height of $A$.

If we represent the relations by graph paper diagrams, we will represent the composition of the relations by arranging the diagram A at the top left, the diagram B at the bottom right and the result $B$ o $A$ at the bottom left, under the diagram $A$ and to the left of diagram $B$.

We thus visualize that the height of $A$ is equal to the width of $B$, the first diagonal making the link, that the width of B o A is equal to the width of A and that the height of $\mathrm{B} \circ \mathrm{A}$ is equal to the height by B.



The relation B o A is the composition of A followed by B
Note that if A and B are everywhere defined and surjective, B o A is also everywhere defined and surjective. Moreover, the composition of relations is associative :

$$
\mathrm{C} \text { o }(\mathrm{B} \text { o } \mathrm{A})=(\mathrm{C} \text { o } \mathrm{B}) \text { o } \mathrm{A} \text { which we will note }: \mathrm{C} \text { o } \mathrm{B} \text { o } \mathrm{A}
$$

The composition of relations is not commutative. We can compose relations in both directions : B o A and $\mathrm{A} \circ \mathrm{B}$, only for square relations and in general B o A is different from $\mathrm{A} \circ \mathrm{B}$. In the practice of weaving we manipulate rectangular diagrams and most time no ambiguity is possible ; however, we will beware of all abusive calculations.

In the calculations it will always be necessary to ensure that the composition of the relations that one writes exists.
B o A exists <=> The width of B is equal to the height of A.
In an equation $\mathrm{A}=\mathrm{B}$ :
we can compose ("multiply") by a relation X to the right of each of the terms of the equation, if and only if the height of X is equal to the width of A and B .
$(A=B=>A \circ X=B \circ X)<=>$ height of $X=$ width of $A$ and $B$
we can compose ("multiply") by a relation X on the left of each of the terms of the equation, if and only if the width of X is equal to the height of A and B .
$(\mathrm{A}=\mathrm{B}=>\mathrm{X}$ o $\mathrm{A}=\mathrm{X} \circ \mathrm{B})<=>$ width of $\mathrm{X}=$ height of A and B
b) Cloth, Peg-plan representation - Threading - Cloth diagram.

The set of diagrams of a cloth represented in the form Peg-plan - Threading - Tissue diagram, can be considered as a relation composition:

Consider the cloth formed from a threading, a peg-plan and a drawdown, the result of the "cloth calculation". Both threading and peg-plan diagrams can be represented by the R and C relations ; the only particularity being that R will always be a mapping. Indeed each end of a threading is threaded in a shaft and only one.
A square of the cloth will be checked if the corresponding warp threads is lifted to this pick. In other words, if we can find a shaft, in which the end is threaded, which is lifted to this pick. The end is in relation with the shaft which itself is related to the pick. The drawdown T is exactly the relation C o R.

Let's look at this in more detail:
A black square in a diagram means that the abscissa is related to the ordinate.
A is the threading: E is the set of ends $\longrightarrow \mathrm{F}$ is the set of shafts
x A y : the end x is threaded in the shaft y
$B$ is the peg-plan: F is the set of shafts $\longrightarrow \mathrm{G}$ is the set of picks y B z: shaft y is lifted at pick z

B o A is the drawdown: E is the set of ends $\longrightarrow \mathrm{G}$ is the set of picks $x$ BoA $z$ : end $x$ is lifted to pick $x$.

| A <br> Threading |  | $\underset{\text { Peg-plan }}{\text { B }}$ |
| :---: | :---: | :---: |
|  |  |  |
| $\mathrm{E} \longrightarrow \mathrm{F} \longrightarrow \mathrm{G}$ |  |  |
| End | Shafts | Picks |
| x | y | z |
|  | B o A drawdown |  |
| E |  | $>\mathrm{G}$ |
| End |  | Picks |
| x |  | z |

Let's translate the definition of the composition of relations into weaving language :
$\forall(x, z) \in \operatorname{ExG} \quad x$ BoA $z \quad<=>\quad \exists y \in F \quad x A y$ and $y B z$
For all x in E and for all z in $\mathrm{G}, \mathrm{x}$ is related to z by the relation BoA, is equivalent to,
there exists an element $y$ of $F$ such that
x is related to y by relation A , and, y is in relation with z by relation B .
For any end x and for any pick z in the BoA drawdown, the end x is lifted to the pick x , is equivalent to,
there is a shaft y such that
the end x is threaded into the shaft y in the threading, and, the shaft y is lifted to the pick z in the peg-plan.

We will therefore represent the calculation of the drawdown $T$, of the threading $R$ and of the pegplan C , by the composition of the relation R followed by the relation C :

$$
\mathrm{T}=\mathrm{C} \circ \mathrm{R}
$$

We will call $\mathrm{T}=\mathrm{C}$ o R the "calculation of the cloth", or the "cloth formula"
xRy and yCz <=> x CoR z


The drawdown T is the composition of the threading R followed by the peg-plan C $\mathrm{T}=\mathrm{Cor}$

To avoid errors in the English translation, the formulas have not been transcribed.
Thus we will speak by default of :

- a threading R. R is the first letter of Rentrage, threading in French.
- a peg-plan C. C is the first letter of Carton, peg-plan in French.
- a drawdown T. T is the first letter of Tissu, drawdown in French.
- a tie-up A. A is the first letter of Attachage, tie-up in French.
- a treadling M. M is the first letter of Marchure, treadling in French.


The Peg-plan-Threading-Drawdown representation

In the "Peg-plan - Threading - Drawdown" representation, each treadle of the peg-plan is connected to the shaft of the threading of the same number.
The peg-plan columns correspond to the threading rows ; the height of the threading is always equal to the width of the peg-plan.

## 5- RECIPROCAL RELATION

Given a relation $A$ from $E$ to $F$, we call the reciprocal relation of $A$, and we note $\mathrm{A}^{-1}$, the relation from F to E such that :

$$
\begin{aligned}
& \mathrm{A}^{-1}=(\mathrm{y}, \mathrm{x}) \in \mathrm{FXE} / \mathrm{xA} \mathrm{y} \\
& \forall(\mathrm{y}, \mathrm{x}) \in \mathrm{FXE} \quad \mathrm{y} \mathrm{~A}^{-1} \mathrm{x}<=>\mathrm{xA} y \\
& \mathrm{E} \xrightarrow[\mathrm{x}]{\mathrm{A}} \mathrm{~F} \\
& \mathrm{E}<\stackrel{\mathrm{A}^{-1} \quad \mathrm{y}}{\mathrm{x}} \mathrm{~F} \\
& \mathrm{y}
\end{aligned}
$$

The diagram of the reciprocal relation $\mathrm{A}^{-1}$ is, the symmetrical of that of $A$, with respect to the first
 bisector.
We go from A to $\mathrm{A}^{-1}$ by exchanging rows and columns.
The width of $\mathrm{A}^{-1}$ is equal to the height of A .
The height of $\mathrm{A}^{-1}$ is equal to the width of A .
Note : the reciprocal of a relation should not be confused with the reciprocal mapping of a bijection. A relation is not in general a mapping and a mapping has a reciprocal mapping only if it is bijective. However, in the case of a bijective relation, the two notions coincide ; we can consider the notion of reciprocal relation as an extension of the notion of reciprocal of a bijection.

It is clear that the reciprocal of the reciprocal of $A$ is equal to $A$ :

$$
\left(\mathrm{A}^{-1}\right)^{-1}=\mathrm{A}
$$

We have already noticed that the notions of mapping and injection are "symmetrical", one applying to columns and the other to rows. Let us express this result more rigorously by showing that if A is injective then $A^{-1}$ is a mapping and vice versa.

A is a mapping from E to F
$\forall x \in E \quad \exists!y \in F \quad x A y$
$\forall x \in E \quad\left(\exists y \in F \quad x A y \quad\right.$ and $\quad \forall\left(y^{\prime}, y^{\prime \prime}\right) \in F^{2} \quad x A y^{\prime} \quad$ and $\left.\quad x A y^{\prime \prime}=>\quad y^{\prime}=y^{\prime \prime}\right)$
$\forall x \in E \quad\left(\forall\left(y^{\prime}, y^{\prime \prime}\right) \in F^{2} \quad x A y^{\prime}\right.$ and $x A y^{\prime \prime} \quad=>y^{\prime}=y^{\prime \prime}$
Because A is everywhere defined $(\forall x \in E \quad \exists y \in F x A y)$
$\forall x \in E \quad\left(\forall\left(y^{\prime}, y^{\prime \prime}\right) \in F^{2} \quad y^{\prime} A^{-1} x \quad\right.$ and $\quad y^{\prime \prime} A^{-1} x \quad=>\quad y^{\prime}=y^{\prime \prime}$
$\mathrm{A}^{-1}$ is an injection from F to E
Applying this result to $\mathrm{A}^{-1}$ we can state :

$$
\begin{array}{lll}
\text { A is a mapping } & <=> & \mathrm{A}^{-1} \text { is an injection } \\
\mathrm{A} \text { is an injection } & <=> & \mathrm{A}^{-1} \text { is a mapping }
\end{array}
$$

## 6- SYMMETRICAL RELATION

We will say of a relation $A$ that it is symmetric if it is equal to its reciprocal : $A=A^{-1}$
$\forall(x, y) \in E x E \quad x A y=>y A x$


A symmetric relation A is square.
Indeed the height of A is equal to the height of $\mathrm{A}^{-1}$, which is equal to the width of A . The diagram of a symmetric relation is symmetric with respect to the first bisector.

## 7- FIRST DIAGONAL I. STRAIGHT DIAGRAM.

For each set $[1, n]$ there exists a particular relation, the identical mapping of $[1, n]$ in $[1, n]$ that we will note $\mathrm{I}_{\mathrm{n}}$, defined by :

$$
\begin{aligned}
& \mathrm{I}_{\mathrm{n}}=\left((\mathrm{x}, \mathrm{y}) \in[1, \mathrm{n}]^{2} / \mathrm{x}=\mathrm{y}\right) \\
& \forall(\mathrm{x}, \mathrm{y}) \in[1, \mathrm{n}]^{2} \quad \mathrm{x} \mathrm{I}_{\mathrm{n}} \mathrm{y} \quad<=>\quad \mathrm{x}=\mathrm{y}
\end{aligned}
$$


$\mathrm{I}_{\mathrm{n}}$, is bijective, square and of side n . When no confusion is possible, it will simply be noted I . The diagram of $I_{n}$ is the first diagonal of the square of side $n$. In weaving we will speak of straight diagram by dimensions $n$.

I is symmetric : $\quad \mathrm{I}=\mathrm{I}^{-1}$

Given a relation A from E to F and I the identical mapping of F ( A and I have the same height) we have :

$$
\text { I o } \mathrm{A}=\mathrm{A}
$$

let $x$ of E and y of F

| x IoA y | <=> | $\exists \mathrm{z} \in \mathrm{F}$ | xAz and z I y |
| :---: | :---: | :---: | :---: |
| x IoA y | $<=>$ | $\exists \mathrm{z} \in \mathrm{F}$ | $x \mathrm{Az}$ and $\mathrm{z}=\mathrm{y}$ |
| x IoA y | <=> | x A y |  |



I o $A=A$

Given a relation $B$ from $E$ to $F$ and I the identical mapping of $E(B$ and I have the same width) we have :

$$
\mathrm{B} \text { o } \mathrm{I}=\mathrm{B}
$$

let $x$ of E and y of F

| $x$ BoIy | $<=>$ | $\exists z \in E$ | $x I z \quad$ and $z B y$ |
| :--- | :--- | :--- | :--- |
| $x$ BoIy | $<=>$ | $\exists z \in E$ | $x=z \quad$ and $\quad z B y$ |
| $x$ BoIy | $<=>$ | $x B y$ |  |


$B$ o $I=B$

We find here the results concerning the weaving :
A straight treadling reproduces the threading to the drawdown.
Straight threading replicates threading to the drawdown.
Although care should always be taken with the size of the diagonal and the direction of the composition of the relations, for the calculation, we will retain that we can simplify by I to the right and to the left.

$$
\text { I o A = A B } \quad \mathrm{B}=\mathrm{B}
$$

## 8- RECIPROCAL OF THE COMPOSITE OF TWO RELATIONS

Given a relation A, from E to F, a relation B from F to G and the composite B o A.

consider the reciprocal ( B o A $)^{-1}$ of the composite B o A

let z be an element of G and x be an element of E

| $\mathrm{z}(\mathrm{BoA})^{-1} \mathrm{x}$ | <=> | xBoAz | definition of ( BooA$)^{-1}$ |  |  | definitions de $\mathrm{A}^{-1}$ and de |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{z}(\mathrm{BoA})^{-1} \mathrm{x}$ | <=> | $\exists \mathrm{y} \in \mathrm{F}$ | $x \mathrm{~A} y$ | and | y B |  |
| $\mathrm{z}(\mathrm{Boa})^{-1} \mathrm{x}$ | <=> | $\exists \mathrm{y} \in \mathrm{F}$ | $\mathrm{y} \mathrm{A}^{-1} \mathrm{x}$ | and | $\mathrm{zB}^{-1} \mathrm{y}$ |  |
| $\mathrm{B}^{-1}$ |  |  |  |  |  |  |
| $\mathrm{z}(\mathrm{Boa})^{-1} \mathrm{x}$ | <=> | $\mathrm{z} \mathrm{A}^{-1} \mathrm{ob}^{-1}$ |  |  |  |  |

So we have

$$
(\mathrm{B} \text { o A })^{-1}=\mathrm{A}^{-1} \text { o } \mathrm{B}^{-1}
$$



B o A

$\mathrm{A}^{-1}$ o $^{-1}=(\mathrm{B} \text { o A })^{-1}$
The reciprocal of the composite is therefore equal to the composition, in the opposite order, of the reciprocals.
This result easily generalizes to the composition of several relations :

$$
(\mathrm{Cob} \text { o o A })^{-1}=\mathrm{A}^{-1} \text { o B }^{-1} \text { o C } \mathrm{C}^{-1}
$$

$(\mathrm{C} \text { o B o A })^{-1}=\left((\mathrm{C} \text { o B) o A })^{-1}\right.$
$(\mathrm{C} \text { о } \mathrm{B} \text { o } \mathrm{A})^{-1}=\mathrm{A}^{-1}$ о $(\mathrm{C} \text { о })^{-1}$
$(\mathrm{C} \text { o B o A })^{-1}=\mathrm{A}^{-1}$ o $\left(\mathrm{B}^{-1}\right.$ o $\left.\mathrm{C}^{-1}\right)$
$(\mathrm{C} \text { о B o A })^{-1}=\mathrm{A}^{-1}$ o $\mathrm{B}^{-1}$ o C-1

## 9- COMPOSITE OF A RELATION WITH ITS RECIPROCAL

a) Properties

Let A be a relation from E to F and $\mathrm{A}^{-1}$ from F to E its reciprocal.
The composite $\mathrm{A}^{-1} \mathrm{o}$ A and the composite A o $\mathrm{A}^{-1}$ still exist, because the width of $\mathrm{A}^{-1}$ is equal to the height of $A$, and the width of $A$ is equal to the height of $\mathrm{A}^{-1}$.

Let us show that $A^{-1}$ o $A$ is symmetric :

$$
\begin{aligned}
& \left(A^{-1} \text { o A }\right)^{-1}=A^{-1} \text { o }\left(A^{-1}\right)^{-1} \\
& \left(A^{-1} \text { o A }\right)^{-1}=A^{-1} \text { o A }
\end{aligned}
$$

$\mathrm{A}^{-1} \mathrm{o} \mathrm{A}$ being equal to its reciprocal, is therefore symmetric.
Let us further show that $\mathrm{A}^{-1} \mathrm{o}$ A contains the diagonal of E :
A is everywhere defined so :

$$
\begin{aligned}
& \forall x \in E \exists y \in F \quad x A y \\
& x A y<=>y^{-1} x
\end{aligned}
$$

So we have

$$
\forall \mathrm{x} \in \mathrm{E} \quad \exists \mathrm{y} \in \mathrm{~F} \quad \mathrm{xA} \mathrm{y} \text { and } \mathrm{y} \mathrm{~A}^{-1} \mathrm{x}
$$

That is to say $\forall x \in E \quad x\left(A^{-1} O A\right) x$
all points $(x, x)$ of the diagonal $I$ of $E$ therefore belong to $\mathrm{A}^{-1} \mathrm{OA}$

These results apply to any relation and in particular to $\mathrm{A}^{-1}$
$\left(\mathrm{A}^{-1}\right)^{-1} \mathrm{o}\left(\mathrm{A}^{-1}\right)$, i.e. A o $\mathrm{A}^{-1}$ is therefore also symmetric.
Moreover A o $\mathrm{A}^{-1}$ contains the diagonal of F .

$\mathrm{A}^{-1} \mathrm{OA}$ and $\mathrm{AoA}^{-1}$ are symmetric and contain their diagonal I (in red) The composite of a relation A followed by its reciprocal is symmetric and contains I
b) "Weaved as drawn in"

Let's anticipate chapter 6 . This situation corresponds to the case where, in weaving, threading is taken as treadling ; we still say that we "weave as drawn in". In fact we take for treadling $R^{-1}$, the reciprocal of threading $R$. The cloth $T$ has the form : $T=R^{-1} o R$

We now know that such a cloth will contain the first bisector and will be symmetric with respect to it. This symmetrical curve organized around the first diagonal is of great importance from the graphical point of view ; Brandon-Guiguet calls it "threading axial".


When the threading is used as treadling (when we weave as drawn in), the graphic line of the cloth contains the first diagonal (in red) and is symmetrical with respect to it.

Note that the phrase "threading is taken as treadling" implies : symmetrically with respect to the first bisector ; treadling is not equal to threading but to its reciprocal. The abilities of a particular threading to produce symmetric graphics by repeating diagonally are highlighted here; we will see later that this is not the only case to consider. The symmetry here concerns the graphic line of the cloth alone ; the tie-up is straight and the treadling is also not set in weave structures because it is the symmetry of a threading. We will also see later under what condition this property is preserved when we set weave structures in the tie-up.
c) Simplification rules. Invertible relations.

We have seen that $\mathrm{A}^{-1} \mathrm{o}$ A contains the diagonal I , i.e. that I is included in $\mathrm{A}^{-1} \mathrm{o} \mathrm{A}$.
Let us now find under which condition $\mathrm{A}^{-1} \mathrm{o} \mathrm{A}$ is equal to I .
Suppose that $\mathrm{A}^{-1} \mathrm{o} \mathrm{A}=\mathrm{I}$; let us show that then A is injective.
Let $x^{\prime}$ and $x^{\prime \prime}$ be elements of $E$ in relation with an element $y$ of $F$ :

$$
\begin{array}{lll}
x^{\prime} A y & \text { and } x^{\prime \prime} A y & => \\
x^{\prime} A y \text { and } y A^{-1} x " \\
x^{\prime} A y \text { and } x^{\prime \prime} A y & => & x^{\prime} A^{-1} O A x^{\prime \prime} \\
x^{\prime} A y \text { and } x^{\prime \prime} A y & => & x^{\prime} I x^{\prime \prime} \\
x^{\prime} A y \text { and } x^{\prime \prime} A y & => & x^{\prime}=x^{\prime \prime}
\end{array}
$$

A is therefore injective
Conversely let us show that if A is injective, then $\mathrm{A}^{-1}$ o $\mathrm{A}=\mathrm{I}$.
Let an element $x$ of E in relation by $\mathrm{A}^{-1} \mathrm{oA}$ with an element z of E
$x A^{-1} O A z \quad=>y \in F \quad x A y$ and $y^{-1} z$
$x A^{-1} \mathrm{oA} z<=>\exists y \in F \quad x A y$ and $z A y$
$\mathrm{x} \mathrm{A}^{-1} \mathrm{OA} \mathrm{z} \quad=>\quad \mathrm{x}=\mathrm{z} \quad$ because A is injective
$\mathrm{x} \mathrm{A}^{-1} \mathrm{oA} \mathrm{z} \quad=>\quad \mathrm{xIz}$
$\mathrm{A}^{-1} \mathrm{o}$ A containing I we therefore have $\mathrm{A}^{-1} \mathrm{o} \mathrm{A}=\mathrm{I}$


We can therefore, when A is injective, simplify by A on the left :

$$
\begin{aligned}
& \text { A oX X A oY }=>A^{-1} \text { o A o X }=A^{-1} \circ \text { A oY } \\
& \text { A oX }=A \circ Y=>I \circ X=I \circ Y \\
& A \circ X=A \circ Y=>X=Y
\end{aligned}
$$

When A is injective, we can simplify by A on the left

$$
\text { A o } \mathrm{X}=\mathrm{A} \circ \mathrm{Y} \text { and } \mathrm{A} \text { is Injective }=>\quad \mathrm{X}=\mathrm{Y}
$$

Applying this result to $\mathrm{A}^{-1}$ we can write :
$\left(\mathrm{A}^{-1}\right)^{-1} \mathrm{o} \mathrm{A}^{-1}=\mathrm{I} \quad<=>\quad \mathrm{A}^{-1}$ is injective
$\mathrm{A}^{-1}$ injective $\quad<=>\quad \mathrm{A}$ is a mapping


A o A ${ }^{-1}=\mathrm{I} \quad<=>\quad \mathrm{A}$ is a mapping
We can therefore, when A is a mapping, simplify by A on the right :

$$
\begin{aligned}
& \mathrm{XoA}=\mathrm{YoA}=>\mathrm{XoAoA}=\mathrm{YoAoA} \\
& \mathrm{XoA}=\mathrm{YoA} \\
& \mathrm{XoA}=\mathrm{YoA}=>X o I=Y o I \\
& \mathrm{X}=\mathrm{Y}
\end{aligned}
$$

$$
\text { When A is a mapping, we can simplify by } \mathrm{A} \text { on the right }
$$

$$
\mathrm{XoA}=\mathrm{YoA} \text { and } \mathrm{A} \text { is a mapping }=>\mathrm{X}=\mathrm{Y}
$$

In the case of a square relation, we can deduce from the two previous results :

$$
\mathrm{A}^{-1} \mathrm{oA}=\mathrm{A}_{\mathrm{ol}} \mathrm{~A}^{-1}=\mathrm{I} \quad<=>\quad \mathrm{A} \text { is a bijection }
$$

On the other hand we can, when A is bijective, simplify by A on the right and on the left :
When A is a bijection, we can simplify by A on the right and on the left

$$
(\mathrm{XoA}=\mathrm{Y} \circ \mathrm{~A} \text { or } \mathrm{A} \circ \mathrm{X}=\mathrm{AoY}) \text { and } \mathrm{A} \text { is a bijection } \Rightarrow \mathrm{X}=\mathrm{Y}
$$

Let's summarize the previous results :
$\mathrm{A}^{-1} \mathrm{o} A=\mathrm{I} \quad<=>\quad \mathrm{A}$ is injective
We can simplify by A on the left
A o $A^{-1}=I \quad<=>\quad$ A is a mapping
We can simplify by A on the right
$\mathrm{A}^{-1}$ o $\mathrm{A}=\mathrm{A}_{\mathrm{ol}} \mathrm{A}^{-1}=\mathrm{I} \quad<=>\quad \mathrm{A}$ is a bijection
We can simplify by A on the right and on the left

If two relations $A$ and $B$ are injective, and the composite $B$ o $A$ exists, then $B$ o $A$ is injective.
Consider two injective relations A and B whose composite B o A exists.

$$
\begin{aligned}
& \mathrm{A}^{-1} \mathrm{o} A=\mathrm{I} \quad<=>\quad \mathrm{A} \text { is injective } \\
& \mathrm{B}^{-1} \mathrm{o} \mathrm{~B}=\mathrm{I} \quad<=>\quad \mathrm{B} \text { is injective }
\end{aligned}
$$

let's calculate $(\mathrm{B} \text { o })^{-1}$ o ( B o A)
$(\mathrm{B} \text { o A) })^{-1}$ o $(\mathrm{BoA})=\mathrm{A}^{-1}$ o $\mathrm{B}^{-1}$ o B o A
$(\mathrm{B} \text { o A })^{-1} \mathrm{o}(\mathrm{B}$ o A) $)=\mathrm{A}^{-1}$ o I o A
$(\mathrm{B} \text { o A })^{-1} \mathrm{o}\left(\mathrm{B}\right.$ o A) $=\mathrm{A}^{-1}$ o A
$(\mathrm{B} \circ \mathrm{A})^{-1} \mathrm{o}(\mathrm{B} \circ \mathrm{A})=\mathrm{I}$
So B o A is injective

$A$ and $B$ injective $=>B$ o $A$ is injective.
In the same way
If two relations $A$ and $B$ are mappings, and the composite $B$ o $A$ exists, then $B$ o $A$ is a mapping.
Consider two mappings A and B whose composite B o A exists.
Then ( B o A) $)^{-1}$ exists ; $\mathrm{A}^{-1}$ o $\mathrm{B}^{-1}$ exists.
A and B are mappings, then $\mathrm{A}^{-1}$ and $\mathrm{B}^{-1}$ are injective.
So $\mathrm{A}^{-1}$ o $\mathrm{B}^{-1}$ is injective.
So $\left(\mathrm{A}^{-1} \text { o } \mathrm{B}^{-1}\right)^{-1}$ is a mapping.
So $\left(\mathrm{B}^{-1}\right)^{-1} \mathrm{o}\left(\mathrm{A}^{-1}\right)^{-1}$ is a mapping.
So B o A is a mapping.


A and B are mappings $=>\mathrm{B}$ o A is a mapping.

We will say that a relation $A$ is invertible if it has an inverse $B$; that is, if there is a relation $B$ such that $\quad \mathrm{A} \circ \mathrm{B}=\mathrm{BoA}=\mathrm{I}$

Remark : if we consider the set of square relations of side n, we can say that, if A is a bijection, then A is invertible and that its inverse is equal to its reciprocal A . This partly justifies the notation of the reciprocal of $\mathrm{A}^{\text {b }} \mathrm{A}^{-1}$. Let us fully justify this notation by showing that all invertible relations are bijections and that their inverses are equal to their reciprocal.

Let A be an invertible relation and B its inverse :
$\mathrm{A} \circ \mathrm{B}=\mathrm{B}$ o $\mathrm{A}=\mathrm{I}$
$A$ and $B$ must therefore be square; they are relations from $E$ to $E$.
We have by definition of I:
$\forall x \in E \quad x I x$
and therefore knowing that B o $\mathrm{A}=\mathrm{I}$ :
$\forall x \in E \quad \exists y \in E x A y$ and $y B x$
We can therefore affirm that A is everywhere defined and that B is surjective. By reasoning with $A$ o $B$ we show that $A$ is surjective and that $B$ is everywhere defined.

Let us show that A is injective :
Let y be an element of $\mathrm{E}, \mathrm{x}^{\prime}$ and x " elements of E such that $\mathrm{x}^{\prime} \mathrm{A}$ y and x " $\mathrm{A} y$
$B$ is everywhere defined so there exists an element $z$ of $E$ such that : $y B z$

$$
\begin{array}{lll}
\mathrm{x}^{\prime} \mathrm{A} \text { a } \text { and } \mathrm{yBz} & => & \mathrm{x}^{\prime} \mathrm{BoAz} \\
\mathrm{x}^{\prime} \mathrm{A} y \text { and } \mathrm{yBz} & => & \mathrm{x}^{\prime \prime z} \\
\mathrm{x}^{\prime} \mathrm{Ay} \text { and } \mathrm{yBz} & => & \mathrm{x}^{\prime}=\mathrm{z}
\end{array}
$$

likewise

$$
\begin{array}{lll}
x^{\prime \prime} A y & \text { and } \mathrm{y} z & => \\
\mathrm{x}^{\prime \prime} \mathrm{A} y \text { and } \mathrm{yBz} & =\mathrm{z} \\
\mathrm{x}^{\prime \prime}=\mathrm{x}
\end{array}
$$

A is therefore injective
A is square and injective, therefore bijective. According to what precedes its inverse B is therefore equal to $\mathrm{A}^{-1}$.

The notion of reciprocal of a relation therefore coincides with the notion of inverse, for bijections. The notation of the reciprocal of a relation $\mathrm{A}^{\text {by }} \mathrm{A}^{-1}$ is therefore acceptable, moreover it prolongs the notion of reciprocal mapping, traditionally noted $\mathrm{f}^{-1}$.

## 10- INVOLUTIVE RELATION

A relation is said to be involutive if it is both symmetric and bijective.
We then have : $\quad \mathrm{A}=\mathrm{A}^{-1} \quad$ and $\quad \mathrm{A}^{-1}$ o $\mathrm{A}=\mathrm{A}_{\mathrm{o}} \mathrm{A}^{-1}=\mathrm{I}$ ie

$$
\mathrm{A}^{2}=\mathrm{I}
$$

Conversely consider a relation A such that $\mathrm{A}^{2}=\mathrm{I}$ :
if we write: A o $\mathrm{A}=\mathrm{A}$ o $\mathrm{A}=\mathrm{I}$
it follows from the above that A is invertible and bijective. Moreover its inverse A is equal to $\mathrm{A}^{-1}$, which means that A is symmetric.

```
A is involutive <=> A = A-1 and A-1 o A = A o A-1 = I
A is involutive <=> A is symmetric and bijective
A is involutive <=> A
```


## 11- SECOND DIAGONAL -I . RETURN DIAGRAM.

For each set $\mathrm{E}=[1, \mathrm{n}]$ : there is a particular relation, which we will note $-\mathrm{I}_{\mathrm{n}}$, which matches each x of E with its opposite modulo $\mathrm{n}+1$, that is $\mathrm{n}+1-\mathrm{x}$. We will denote this relation -I when there is no ambiguity.

$$
\forall(x, y) \in[1, n]^{2} \quad x-I_{n} y \quad<=>x+y=n+1
$$



The diagram of $-\mathrm{I}_{\mathrm{n}}$ is the second diagonal of the square of side n . In weaving we will speak of return diagram.
$-I_{n}$ is bijective, symmetric and therefore involutive.
$(-\mathrm{I})^{-1}=-\mathrm{I} \quad$ and $\quad(-\mathrm{I})^{-2}=\mathrm{I}$

## 12- CALCULATION ON STRAIGHT AND RETURN DIAGRAMS

Here we find the calculation rules on straight and return diagrams :


A simple mnemonic is to think of the rule of signs : minus by minus equals plus, etc., the + representing the first diagonal I (rising to the right) and the - representing the second diagonal (rising to the left). In a straight-and-return weave, isolating one straight or return repeat, from the threading and another in the treadling one is faced with the computation of one of the above elementary weaves. With these rules in mind, you can very quickly establish a complete diagram of the cloth (like those of Rondo Amigo).
This technique can of course be extended to threading and treadling by straight and return blocks. However if here I commutes with -I, as with all relations, it is not the same for $-I$ as we will see in what follows. The analogy with the signs is therefore to be restricted to this very specific case.
a) Expression of a geometric transformation

Composing a relation A with -I can be seen as subjecting it to a simple geometric transformation :


Note that the -I relation in the -I o A expression is not the same as the -I relation in the A o -I expression.
In the expression -I o A , -I has the same width as the height of A.
In the expression A o -I , -I has the same height as the width of A.
b) Expression of any geometric transformation

In weaving, complex designs are most often constructed using an elementary pattern that has been turned or reversed a certain number of times ; in other words, the basic pattern is subjected to a simple geometric transformation. Let us clarify this point : the pattern is not deformed, the geometric transformation is such that the transformed pattern is "superimposable" on the starting pattern. The drawings are made on squared paper. When we talk about turning a pattern, we mean turning it one or more "quarter turns", in one direction or the other. When we talk about reversing a pattern, it is, of course, vertically or horizontally.

These simple geometric transformations, or positions of a diagram, are eight in number. They can all be deduced from the composition of a symmetry and a rotation. They can also all be deduced from the composition of the symmetry with respect to the first diagonal (a transformation which makes pass from a relation A to its reciprocal), and a symmetry.


+ for a rotation means: counterclockwise.

Although it is legitimate to describe the interaction of all these geometric transformations and the composition of the relations (and therefore the drawdown), by not privileging any of them, we will limit ourselves to views so far. Indeed the results relating to all the transformations are too numerous to be all retained, moreover it would be necessary to introduce a new notation more homogeneous of these transformations.

A particular transformation will therefore be deduced simply from the three preceding ones. Note for example :

The diagram $\mathrm{A}^{-1} \mathrm{o}-\mathrm{I}=(-\mathrm{I} \text { o } \mathrm{A})^{-1}$ is equal to the diagram of A rotated by $+90^{\circ}$.
The diagram -I o $\mathrm{A}^{-1}=(\mathrm{A} \circ-\mathrm{I})^{-1}$ is equal to the diagram of A rotated by $-90^{\circ}$.
The diagram -I o A o $-\mathrm{I}=\left(-\mathrm{I} \text { o } \mathrm{A}^{-1} \text { o }-\mathrm{I}\right)^{-1}$ is equal to the diagram of A rotated by $180^{\circ}$.
c) Consequences of a symmetry on the threading or on the peg-plan (peg-plan-threading-drawdown representation).

Consider a cloth of type peg-plan - threading $\mathrm{T}_{1}=\mathrm{C}_{1}$ o $\mathrm{R}_{1}$


$$
\mathrm{T}_{1}=\mathrm{C}_{1} \circ \mathrm{R}_{1}
$$

If we replace threading $\mathrm{R}_{1}$ with its symmetric with respect to the vertical $\mathrm{R}_{2}=$ $\mathrm{R}_{1}$ o - I , the new cloth $\mathrm{T}_{2}=\mathrm{C}_{1}$ o $\mathrm{R}_{2}$ will be the symmetric of $\mathrm{T}_{1}$ with respect to the vertical.

## $\mathrm{R}_{2}=\mathrm{R}_{1} \mathrm{o}-\mathrm{I}$

$\mathrm{R}_{2}$ is the symmetric of $\mathrm{R}_{1}$ with respect to the vertical.
$\mathrm{T}_{2}=\mathrm{C}_{1} \mathrm{o}\left(\mathrm{R}_{1} \mathrm{o}-\mathrm{I}\right)$
$\mathrm{T}_{2}=\left(\mathrm{C}_{1} \circ \mathrm{R}_{1}\right)$ o- I
$\mathrm{T}_{2}=\mathrm{T}_{1}$ o- I
$\mathrm{T}_{2}$ is the symmetric of $\mathrm{T}_{1}$ with respect to the vertical.

$\mathrm{T}_{2}=\mathrm{C}_{1}$ o $\mathrm{R}_{2}=\mathrm{C}_{1}$ o $\left(\mathrm{R}_{1}\right.$ o-I $)=\left(\mathrm{C}_{1}\right.$ o $\left.\mathrm{R}_{1}\right)$ o- $\mathrm{I}=\mathrm{T}_{1}$ o- -I

If we replace the peg-plan $\mathrm{C}_{1}$ by its symmetric with respect to the horizontal $\mathrm{C}_{3}=-\mathrm{I}$ o $\mathrm{C}_{1}$, the new cloth $\mathrm{T}_{3}=\mathrm{C}_{3}$ o $\mathrm{R}_{1}$ will be the symmetric of $\mathrm{T}_{1}$ with respect to the horizontal.

$\mathrm{T}_{3}=\mathrm{C}_{3} \circ \mathrm{R}_{1}=\left(-\mathrm{I}\right.$ o $\left.\mathrm{C}_{1}\right)$ o $\mathrm{R}_{1}=-\mathrm{I}$ o $\left(\mathrm{C}_{1} \circ \mathrm{or}_{1}\right)=-\mathrm{I}$ o $\mathrm{T}_{1}$
Rather than memorizing these results, it would be more profitable to know how to find them quickly by calculating the cloth. Once assimilated on simple examples, the calculation of the cloth will be of great service to you for analyzing more complex situation. In addition, you can be satisfied not to have read these rebarbative pages for nothing !

Agree those who number shafts top to bottom and treadles right to left with others:
$\mathrm{R}_{4}=-\mathrm{I}$ o $\mathrm{R}_{1}$
$\mathrm{C}_{4}=\mathrm{C}_{1} \mathrm{o}-\mathrm{I}$
$\mathrm{T}_{4}=\mathrm{C}_{4} \circ \mathrm{R}_{4}$
$\mathrm{T}_{4}=\left(\mathrm{C}_{1} \circ-\mathrm{I}\right)$ o $\left(-\mathrm{I}\right.$ o $\left.\mathrm{R}_{1}\right)$
$\mathrm{T}_{4}=\mathrm{C}_{1}$ o I o $\mathrm{R}_{1}$
$\mathrm{T}_{4}=\mathrm{C}_{1}$ o $\mathrm{R}_{1}$
$\mathrm{T}_{4}=\mathrm{T}_{1}$


$$
\mathrm{T}_{4}=\mathrm{C}_{4} \circ \mathrm{R}_{4}=\left(\mathrm{C}_{1} \circ-\mathrm{I}\right) \circ\left(-\mathrm{I} \circ \mathrm{R}_{1}\right)=\mathrm{C}_{1} \circ \mathrm{R}_{1}=\mathrm{T}_{1}
$$

As an exercise, will you be able to describe these "flipped" diagrams with the help of a judicious calculation?


## Attention

not all symmetry is good for the cloth !
Touching the correspondence between the shafts and the treadles is often fatal for the cloth. Also note in passing the importance of the order of the diagram composition calculations, this "multiplication" is not commutative.
here :
$\mathrm{R}_{4}=-\mathrm{I}$ o $\mathrm{R}_{1}$
$\mathrm{T}_{5}=\mathrm{C}_{1}$ o $\mathrm{R}_{4}$
$\mathrm{T}_{5}=\mathrm{C}_{1}$ o (-I o $\left.\mathrm{R}_{1}\right)$
and
$\mathrm{T}_{5} \neq \mathrm{T}_{1}$

$\underline{\text { https://oliviermasson.art }{ }^{\circledR}}$
chapter 3
representation of the cloth using matrices.

We will again look at the cloth through a new mathematical lens, that of matrices. This view is very close to the previous one and we will not dwell on it. However, it will allow us to introduce some notions that were still missing in our building.

## 1- DEFINITIONS

We can directly consider a relation diagram as a matrix by specifying the following points:

- The matrices are defined on the set which contains the two elements 0 and 1 , equipped with the two operations "or" and "and" (noted " v " and " $\wedge$ ")
A checked square (black) will be marked 1
An empty square (white) will be marked 0
- The lines of a relation will be numbered in the opposite direction to the standard direction for matrices, from bottom to top.

Representation of a diagram by a relation


$$
A(n, p)
$$

Representation of a diagram by a matrix A ( $n, p$ ), n columns and $p$ rows.

The element of column i and row j will be denoted aj.

$$
\mathrm{F}=[1, \mathrm{p}]\left(\begin{array}{cccccccccccccccccccc}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0_{j} & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & k_{\text {an }} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{array}\right)
$$

123
i
n

$$
E=[1, n]
$$

All the notions defined for the relations can be transcribed in the language of matrices :

Identical relation
Reciprocal of a relation
Symmetric relation

Unity matrix (inverted).
Transpose of a matrix.
Symmetric matrix

## 2- PRODUCT OF MATRICES

The most interesting parallel remains the equivalence of the composition of relations and the product of matrices :

Product of matrices

$$
\mathrm{k}=1
$$



$$
\begin{aligned}
& \mathrm{n}\left(\begin{array}{ccccccccccccccccc} 
\\
0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{i}^{\mathrm{k}} & 0 & 1 & 0 \\
1 & & & & & & & & & & & & i & & & \mathrm{~m}
\end{array}\right) \mathrm{k} \\
& \left(\begin{array}{lllllllllllllllll}
1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\
0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\
0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
\end{array}\right)\left(\begin{array}{lllll}
1 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 1 \\
0 & 1 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0
\end{array}\right) \boldsymbol{B} \\
& \left(\left.\begin{array}{lllllllllllllllll}
1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & c_{i}^{j} & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\
0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
1 & & & & & & & & & & & i & & & m
\end{array} \right\rvert\, \begin{array}{ccccc}
\mathrm{b}_{\mathrm{k}}^{j} & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 \\
\mathrm{k} & & & & \mathrm{n}
\end{array}\right)
\end{aligned}
$$

## 3- LOGICAL OPERATIONS ON RELATION

The first interest of this representation is to show the logical operations carried out on the squares of the diagram for the calculation of the cloth. These logical operations are described, in the composition of the relations, by the different quantifiers, but in a global way. Note also that it is a calculation of this type that is actually performed by the computer to display a cloth.

The second is to allow us to define, in a natural way, logical operators on the diagrams :
a) "Not A " relation

Given a diagram $\mathrm{A}=\left(\mathrm{a}_{\mathrm{i}}\right)^{\prime}$, we define the diagram $\operatorname{Not} \mathrm{A}=\left(\mathrm{a}_{\mathrm{i}} \mathrm{j}\right)$, which we denote $\neg \mathrm{A}$ :

$$
\forall(i, j) \in N \times P \quad a^{\prime} j=\neg a_{j}^{j}
$$



A

$\neg \mathrm{A}$

1s become 0s and vice versa.

$$
\neg 0=1
$$

and $ᄀ 1=0$
Full (black) squares become empty (white) and vice versa.
b) Relation "A or B"

Given two diagrams $\mathrm{A}=\left(\mathrm{a}_{\mathrm{i}}\right)$ and $\mathrm{B}=\left(\mathrm{b}_{\mathbf{i}}\right)$ of the same size, we define the diagram "A or B " $=\left(\mathrm{c}_{\mathrm{i}}^{\mathrm{j}}\right)$, which we note $\mathrm{A} \vee \mathrm{B}$ :

$$
\forall(i, j) \in N \times P \quad c_{i}^{j}=a_{j}^{j} \text { or } b_{i j}^{j}=a_{i j}^{j} v b_{i j}^{j}
$$



The diagram "A or $B$ " is obtained by superimposing the diagram of $A$ on that of $B$, each square checked, in A or in B , will be checked in " A or B "
c) Relation "A and B"

Given two diagrams $\mathrm{A}=\left(\mathrm{a}_{\mathrm{i}}\right)$ and $\mathrm{B}=\left(\mathrm{b}_{\mathrm{i}}\right)$ of the same size, we define the diagram
" A and $\mathrm{B} "=\left(\mathrm{c}_{\mathrm{i}}\right)^{2}$, which we note $\mathrm{A} \wedge \mathrm{B}$ :

$$
\forall(i, j) \in N \times P \quad c_{i}^{j}=a_{i j}^{j} \text { and } b_{j}^{j}=a_{j}^{j} \wedge b_{i}^{j}
$$



The "A and $B$ " diagram is obtained by superimposing the diagram of $A$ on that of $B$, each square checked, both in A and in B, will be checked "A and B".

## C "TREADLING - TIE-UP - THREADING - DRAWDOWN" REPRESENTATION

We are now going to approach the representation of the cloth by a diagram comprising four elements : the threading, the tie-up, the treadling and the drawdown. Remember that the main interest of using the tie-up is to make it possible to separate different aspects of a cloth, such as its purely graphic qualities from its weave structure characteristics. This type of cloth therefore allows a complete study of weave structures. However, we will still have to add the color diagrams to have an overall view of the cloth.
chapter 1
definition

## 1- CLOTH DIAGRAM FORMULA

In the first representation of the cloth, of the "peg-plan-threading-drawdown" type, each treadle actuating a shaft and only one : there is a direct correspondence between the treadles and the shafts. In the "threading-tie-up-treadling-drawdown" representation there are two new diagrams : the tie-up and the treadling. The treadling features treadles, which are attached to countermarche, as shown in the tie-up, with each of the countermarche commanding a shaft. Adding a tie-up is a bit like placing an intermediary between treadling, which plays the role of a pre-peg-plan, and threading :


Each treadle, each column of the treadling, is "attached" to one or more contremarches, to one or more rows of the tie-up. The tie-up diagram indicates the correspondence of each treadle of the treadling with one or more contremarches which each actuate a shaft.

To find out which shafts are lifted at a certain pick, just look in the treadling which treadles are actuated, then read in the tie-up which shafts are controlled by each of these treadles. This approach strangely resembles a calculation of cloth, the tie-up playing the role of threading !


For the pick z , the treadle is pressed there, to actuate (among other things) the contermarche w, which raises the shaft w , which raises (among other things) the end x to the pick z .

x $\mathrm{MoA}^{-1} \mathrm{oR} \mathrm{z}$
because y Aw $<=>$ w A $^{-1}$ y Note the intermediate variables

We therefore obtain the formula of the cloth in the representation "threading-tie-up-treadlingdrawdown" (the formula of the cloth with tie-up) :

$$
\mathrm{T}=\mathrm{Mo} \mathrm{~A} \mathrm{~A}^{-1} \mathrm{o} \mathrm{R}
$$

This cloth formula with tie-up $\mathrm{T}=\mathrm{M} \mathrm{o}^{-1} \mathrm{o} \mathrm{R}$, can be seen as two simple successive calculations:
First calculation, the calculation of the peg-plan $\mathrm{C}=\left(\mathrm{M} \mathrm{o} \mathrm{A}^{-1}\right)$
Second calculation, the calculation of the cloth $\mathrm{T}=\mathrm{C}$ o R

$$
\begin{array}{ll}
\mathrm{T}=\left(\mathrm{MoA}^{-1}\right) \text { o } \mathrm{R} & \begin{array}{l}
\text { The calculation of the simple cloth, in peg-plan-threading } \\
\text { representation, with as peg-plan } \mathrm{C}=\left(\mathrm{Mo} \mathrm{~A}^{-1}\right)
\end{array} \\
\mathrm{T}=\mathrm{M} \mathrm{o} \mathrm{~A}^{-1} \text { o } \mathrm{R} & \text { The calculation of the cloth with tie-up }
\end{array}
$$

First calculation:
To get the peg-plan, i.e. the diagram where are noted at each pick (on each line), the lifted shafts, it is enough to calculate the cloth including, the treadling as peg-plan, and, the reciprocal of the tie-up as threading.
$\mathrm{C}=\left(\mathrm{Mo} \mathrm{A}^{-1}\right)$
The peg-plan C the composite of the reciprocal of the tie-up $\mathrm{A}^{-1}$, followed by the treadling M.

Second calculation:
The calculation of the cloth
$\mathrm{T}=\mathrm{C}$ o R
with the peg-plan C result of the previous calculation $\mathrm{C}=(\mathrm{Mo}$ $\mathrm{A}^{-1}$ )
$\mathrm{T}=\mathrm{Co}$ or
$\mathrm{T}=\left(\mathrm{Mo}^{-1}\right) \mathrm{oR}$
$\mathrm{T}=\mathrm{MoA}^{-1} \mathrm{oR}$


The cloth in the "threading-tie-up-treadling-drawdown" representation
$\mathrm{T}=\mathrm{MoA}^{-1} \mathrm{o} \mathrm{R}$
With in addition, on the left of the treadling, the peg-plan C, result of the intermediate calculation $\mathrm{C}=\left(\mathrm{Mo} \mathrm{A}^{-1}\right)$


$$
\mathrm{T}=\mathrm{Mof} \mathrm{~A}^{-1} \mathrm{o} \mathrm{R}
$$

This simple formula will be the basis of our study of weave structures.
The simplified diagram of a cloth is easy to remember and will allow you to quickly find the properties of the diagram of the cloth ; it is also a simple analysis tool that will allow you to find your bearings in more complex situations.
Note again that despite the position of the tie-up A in the complete diagram of the cloth, it is its reciprocal $\mathrm{A}^{-1}$ which intervenes in the calculation.
Note once again that although the notation of the calculus is multiplicative, the composition of the diagrams is not commutative, the order in which the diagrams are written is fundamental.

## 2- COMPATIBILITY OF REPRESENTATIONS <br> 2- COMPATBLLI OF REPRESENTATIONS

The simple cloth representation using the peg-plan and threading can be considered a complete representation if one considers that one is unknowingly using a straight tie-up I :

$\mathrm{T}=\mathrm{MoI}^{-1} \mathrm{o}$ R

$\mathrm{T}=\mathrm{MoR}$

Let's calculate the complete cloth
$\mathrm{T}=\mathrm{MoI} \mathrm{I}^{-1}$ o R
$\mathrm{T}=\mathrm{M}$ oI o $\mathrm{R} \quad$ because we know that $\mathrm{I}^{-1}=\mathrm{I}$
$\mathrm{T}=\mathrm{MoR} \quad$ because we can simplify by I
The diagonal that we draw to ensure the correspondence between the treadles and the shafts can therefore be considered as a diagram in its own right of the cloth, it is its tie-up, it is straight.
From now on we will no longer speak of cloth with two or three elements, we will consider that a drawdown always contains a threading, a tie-up and a treadling. We will specify if necessary if the tie-up is straight.
chapter 2
Change from "treadling-tie-up-threading" representation to "peg-plan-threading" representation Multiple cloth diagram

When the tie-up is not straight, the peg-plan does not appear in the full cloth diagram. However, if a dobby is used, a representation of the peg-plan is necessary.
Previously we saw that to obtain the peg-plan C it sufficed to calculate M o A-1 ; the cloth can then be presented in the form $\mathrm{T}=\mathrm{C}$ o R :

$\mathrm{T}=\mathrm{Cor}$

$\mathrm{C}=\left(\mathrm{Mo} \mathrm{A}^{-1}\right)$

The diagram of the peg-plan is common to these two diagrams. In the first, on the right, it appears as a result, in the second, on the left, it participates in the calculation of the drawdown T. We are going to assemble these two diagrams into a single one :


The peg-plan $\mathrm{C}=\mathrm{M} \mathrm{o} \mathrm{A}^{-1}$ is drawn only once, it participates in two different calculations.

If we were able to show two calculations on the same diagram, why not three !
To have a vision of the cloth in all its aspects, it would also be necessary to be able to read on the same diagram the starting diagram of the cloth : T $=\mathrm{Mo} \mathrm{A}^{-1} \mathrm{o} \mathrm{R}$

The free square, at the top right, of the general diagram seems to have been reserved for tie-up A ! A third calculation can then be read, that of the cloth with tie-up : T $=\mathrm{M} \mathrm{o} \mathrm{A}^{-1}$ o R , considering only the squares forming the corners of the general diagram.


We will call such a diagram, where several cloth diagrams can be read, a cloth multiple diagram. On a cloth multiple diagram, an arrow will indicate the direction of a particular cloth calculation ; the arrow will start from the square playing the role of the treadling, will turn in the "tie-up square" in question, towards the "threading square" to end on the result of the calculation in the "drawdown square". According to one calculation or another, the same square can play a different role in turn ; in our example the peg-plan is either a cloth square or a treadling square.
The arrows will indicate a correct calculation. Indeed with a multiple diagram of 9 squares, like the one we are studying, we could read as many calculations as there are groups of four squares forming the four corners of a rectangle. All these calculations have no a priori reason to be correct ; therefore, caution should be exercised when reading a multiple cloth diagram.

Let's take a closer look at this first example :


Under the threading remains an empty square, let's fill it in by calculating the cloth including the four squares at the top and to the left : $\mathrm{A}^{-1}$ o I o R, i.e. $\mathrm{A}^{-1}$ o R. A third way to obtain our cloth T appears, doing the calculation with the two lower squares of the right column and the two lower squares of the left column: $\mathrm{MoI}^{-1} \mathrm{o}\left(\mathrm{A}^{-1} \mathrm{o} \mathrm{R}\right)=\mathrm{Mo} \mathrm{A}{ }^{-1} \mathrm{o} \mathrm{R}$. This calculation is correct, we find as a result our cloth T.

With the top right squares we have another correct calculation: I o $\mathrm{A}^{-1} \mathrm{o} \mathrm{I}=\mathrm{A}^{-1}$. As with the top two squares in the right column and the top two squares in the left column: Io $\mathrm{A}^{-1} o \mathrm{o}=\mathrm{A}^{-1} \mathrm{o} \mathrm{R}$. With the two squares on the right of the top line and the two squares on the right of the bottom line we find another way of calculating the peg-plan : $\mathrm{M} \mathrm{o}^{-1}$ o $\mathrm{I}=\mathrm{M} \mathrm{o} \mathrm{A}^{-1}$.

In fact only one calculation is wrong, the one made with the four squares at the bottom and on the left : ( M o $\mathrm{A}^{-1}$ ) o ( $\left.\mathrm{A}^{-1}\right)^{-1}$ o ( $\mathrm{A}^{-1}$ o R)
By expanding we obtain: $\mathrm{MoA}^{-1}$ o $\mathrm{A} \mathrm{o}^{-1}$ o R which is in general different from $\mathrm{T}=\mathrm{MoA} \mathrm{A}^{-1} \mathrm{o} \mathrm{R}$. Note that according to the rules established previously, it would suffice for A to be injective or a mapping for the result to be true ; we would then have either $\mathrm{A}^{-1}$ o $\mathrm{A}=\mathrm{I}$ or A o $\mathrm{A}^{-1}=\mathrm{I}$.
Don't worry, we don't necessarily have to check all the possible calculations in a multiple diagram, we will most often focus on two or three main clothes.

Let's restate the initial problem : we are looking for a way to go from a representation of a cloth with tie-up to a representation of the type peg-plan - straight tie-up. We have seen that it was possible to achieve this by having the computer perform two successive cloth calculations. Then we grouped these calculations on a single diagram... why not have the computer make a single calculation ! This is what we are going to do, thus highlighting the very practical interest of cloth multiple diagrams.
Let's artificially compose a composite threading formed by the juxtaposition of threading R and a straight repeat I ; in the same way let us form a composite treadling formed of the treadling M on which we will place a straight repeat I; take A as a tie-up and calculate this composite cloth. We directly obtain the multiple diagram.
The computer does not make a difference between each of the parts of treadling and threading, it calculates globally but always with the cloth algorithm, the calculations actually performed are therefore always correct.


To locate the calculation actually carried out by the computer, we will draw with thicker lines the two axes of separation of the threading of the tie-up and the treadling. The correct calculations are therefore those which use the tie-up in the upper right corner, part of the composite treadling and part of the composite threading ; or those that involve part of the right column and part of the top line. Other calculations made from the "result" squares (of the real calculation) are also true, here the
diagram in the form peg-plan - straight tie-up, although they are not carried out by the computer, otherwise the multiple diagram would not be of interest.
The cloth multiple diagram is therefore also a new calculation tool that will allow us to automatically switch from one cloth to another cloth ; here the diagram of the peg-plan - straight tie-up cloth was calculated automatically from the diagram with tie-up. This tool is very powerful and we will use it extensively in the following, especially for telescoping.

In fact, for our specific problem, the multiple diagram used is too complex. Indeed we do not need to know the incidence of the tie-up on the threading. By deleting the line of squares in the middle we will obtain the multiple type diagram : transition from the representation with tie-up to the peg-plan - straight tie-up representation.


By adding a straight repeat in the threading, we obtain, in a single calculation, the cloth in the form peg-plan - straight tie-up (Pointcarré allows to display the peg-plan or not).

The "cloth multiple diagram" tool of which we have just studied a complete example will be of great help to us for our subsequent research. The interest of this chapter thus largely exceeds the particular problem of the passage from one representation of the cloth to another.
We will systematically use cloth multiple diagrams in the second part of this book.

## D FIRST PRACTICAL CONSEQUENCES OF THE CLOTH FORMULA

The mathematical model that we have just built will initially allow us to take a step back from weaving. Seen from a new perspective, certain properties of the cloth, confusedly felt, will seem clearer, certain notions will be clarified. Secondly, the rigorous mastery of the cloth will allow us to go further in the theory of weaving. New manipulations of the different diagrams will be possible, general methods for putting complex curves into context can be developed.
Let's start by looking at some simple cloth situations, using this new cloth formula projector : $\mathrm{T}=\mathrm{Mo} \mathrm{A}^{-1} \mathrm{o} \mathrm{R}$
chapter 1
another presentation of the cloth diagram
Some authors note a cloth by presenting the peg-plan as an extension of the threading. Let's see how to interpret this representation in our usual diagram.


We have seen that, apart from the threading, the information concerning the cloth is divided between the tie-up and the treadling. In the case of a representation of the threading-straight tie-up-peg-plan type, this information is concentrated in the peg-plan, the tie-up being reduced to I. In contrast, the representation we are dealing with concentrates all this information in the tie-up, the treadling being reduced to I . The tie-up is then equal to $\mathrm{C}^{-1}$, ie. to the reciprocal of the peg-plan.
The interest of this representation is obvious : to be able to note a cloth on a sheet which is wider than it is high !

## chapter 2

geometric transformations of a diagram
The cloth formula $\mathrm{T}=\mathrm{Mo} \mathrm{A}{ }^{-1} \mathrm{o} \mathrm{R}$ allows us to quickly generalize the results already obtained（ First part B12）to cloth with tie－up．A rectangular tie－up can be subjected to a symmetry with respect to the horizontal，or a symmetry with respect to the vertical，or a rotation of $180^{\circ}$ ．On the condition of applying a judicious symmetry on the threading or the treadling or both，we can ensure the invariance of the cloth

Consider a cloth of the＂treadling－tie－up－threading＂type．


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面 5


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सpmpNom


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If we perform a symmetry/vertical simultaneously on the tie-up and on the treadling, the drawdown remains unchanged.


Mo-I is the symmetric of M with respect to the vertical.
A o-I is the symmetric of A with respect to the vertical.
$\mathrm{T}^{\prime}=\left(\mathrm{M}\right.$ o -I) o (A o -I) ${ }^{-1}$ o R
$\mathrm{T}^{\prime}=(\mathrm{Mo-I}) \mathrm{o}(-\mathrm{I})^{-1}$ o $\mathrm{A}^{-1}$ o R
$\mathrm{T}^{\prime}=\mathrm{Mo-I}$ o-I o $\mathrm{A}^{-1}$ o R
$\mathrm{T}^{\prime}=\mathrm{M}$ o I o $\mathrm{A}^{-1}$ o R
$\mathrm{T}^{\prime}=\mathrm{M}$ o $\mathrm{A}^{-1}$ o R
$\mathrm{T}^{\prime}=\mathrm{T}$

If we perform a symmetry/horizontal simultaneously on the tie-up and on the threading, the drawdown remains unchanged.

-I o R is the symmetric of R with respect to the horizontal.
-I o A is the symmetric de A with respect to the horizontal.
$\mathrm{T}^{\prime}=\mathrm{M}$ o $(-\mathrm{I} \circ \mathrm{o})^{-1}$ o (-I o R $)$
$\mathrm{T}^{\prime}=\mathrm{M}$ o $\mathrm{A}^{-1}$ o (-I) ${ }^{-1}$ o (-I o R)
$\mathrm{T}^{\prime}=\mathrm{M}$ o $\mathrm{A}^{-1}$ o -I o -I o R
$\mathrm{T}^{\prime}=\mathrm{M}$ o $\mathrm{A}^{-1}$ o I o R
$\mathrm{T}^{\prime}=\mathrm{Mo} \mathrm{A}^{-1}$ o R
$\mathrm{T}^{\prime}=\mathrm{T}$

If we simultaneously perform a symmetry/horizontal on the threading, a symmetry/vertical on the treadling and a $180^{\circ}$ rotation on the tie-up, the drawdown remains unchanged.


Mo-I is the symmetric of M with respect to the vertical.
-I o R is the symmetric of R with respect to the horizontal.
-I o A o -I is the rotation of A by $180^{\circ}$.
$\mathrm{T}^{\prime}=(\mathrm{M}$ o -I$)$ o $(-\mathrm{I} \text { o } \mathrm{A} \text { o }-\mathrm{I})^{-1}$ o (-I o R)
$\mathrm{T}^{\prime}=(\mathrm{M} \circ-\mathrm{I})$ o $(-\mathrm{I})^{-1}$ o $\mathrm{A}^{-1}$ o (-I) $)^{-1}$ o ( $(\mathrm{I} \quad$ o R $)$
$\mathrm{T}^{\prime}=\mathrm{M}$ o -I o -I o A-1 o -I o -I o R
$\mathrm{T}^{\prime}=\mathrm{M}$ o I o $\mathrm{A}^{-1}$ o I o R
$\mathrm{T}^{\prime}=\mathrm{M}$ o $\mathrm{A}^{-1}$ o R
$\mathrm{T}^{\prime}=\mathrm{T}$

Many other cloth transformations are possible. In particular, the results already obtained, those which did not involve the tie-up, are still valid. With a square tie-up new possibilities of transformation appear. The cloth formula and the calculation on the composition of relations should now allow you to simply analyze any situation.
chapter 3
warp and weft reversal of a cloth
When a treadling has the property of reverse threading, i.e. when it is injective (one cross and only one per line), it may be interesting to completely reverse the cloth $T=\mathrm{Mo} \mathrm{A}^{-1}$ o R , replacing the threading by the reciprocal of the treadling, the treadling by the reciprocal threading and tie-up by its reciprocal.


The new cloth obtained $\mathrm{T}^{\prime}$ is the reciprocal of the cloth T , that is $\mathrm{T}^{-1}$ :

$$
\mathrm{T}^{\prime}=\mathrm{R}^{-1} \mathrm{o}\left(\mathrm{~A}^{-1}\right)^{-1} \circ \mathrm{M}^{-1}=\left(\mathrm{Mo} \mathrm{~A} \mathrm{~A}^{-1} \mathrm{o} \mathrm{R}\right)^{-1}=\mathrm{T}^{-1}
$$

Take part in this disturbing experience : take a sheet of paper, on which a cloth is drawn, by the top right corner and by the bottom left corner. Rotate the sheet and observe the new cloth by transparency...

Let's take the example of an in weave structures cloth (The weaves used are described in the third part, block method).


This traditional cloth, of "overshot" type, is woven on 6 shafts and 8 treadles. Two wefts are used : a thin one of the same color as the warp, for the tabby binding, and a thicker one, of a different color, for the pattern. The weave structure is represented at the top left, at the bottom right is the cloth with its colors. The major drawback of this cloth is that it requires the use of two shuttles ; the weaving is correspondingly slowed down. In addition, its weft density is high. If we reverse the diagram of the cloth, we will have a threading on 8 shafts with two warps and a treadling with only one kind of weft. This cloth will look the same as the previous one, it has two more shafts, but is much faster to weave. In addition, its weft density is lower.


$$
\mathrm{T}^{-1}=\mathrm{R}^{-1} \circ\left(\mathrm{~A}^{-1}\right)^{-1} \circ \mathrm{M}^{-1}=\left(\mathrm{Mo} \mathrm{~A}^{-1} \circ \mathrm{R}\right)^{-1}=\mathrm{T}^{-1}
$$

The structure of the new threading is of the "binding body plus decor body" type ; it is a "summer and winter" threading. This stunning experience may have allowed you to realize that with a "summer and winter" type threading, you can also weave a warp effect overshot.

## Noticed :

$\mathrm{T}^{-1}$ is not exactly the T cloth whose warp and weft have been exchanged.
There is indeed an exchange of rows and columns. Yet a white line of T, that is to say a weft float of $T$, is transformed into a white line of $\mathrm{T}^{-1}$, which is not a warp float of $\mathrm{T}^{-1}$ but a reverse warp float. In the same way a warp float of T is transformed into a reverse weft float of $\mathrm{T}^{-1}$.
The cloth T whose warp and weft have been exchanged is the reverse side of the cloth $\mathrm{T}^{-1}$.
chapter 4
Zoom on the tie-up
Imagine a cloth where all the information is concentrated in the tie-up, the threading and the treadling being straignt.


The cloth T is then exactly equal to the reciprocal of the tie-up $\mathrm{A}^{-1}$.


If we stretch the diagonal I in a rectangular threading and treadling, the cloth will be stretched by the same amount and present an inverted magnification of the tie-up.
Once the enlargement of the shape obtained, we will move on to the setting in weave structure. This type of enlargement retains the definition of the tie-up design and the enlarged cloth will always have a staircase outline.

## chapter 5

generated drawdown

What is the approach of the textile designer ? He starts from a graphic idea, a curve or a surface, then tries to build a threading and a peg-plan likely to produce the desired design on the drawdown. What does the computer do? Completely the opposite! It calculates the drawdown according to the information contained in the peg-plan and in the threading.
Why not turn the situation in our favor ?
The calculation of the cloth in the representation of the "peg-plan-threading" type is expressed by the formula $\mathrm{T}=\mathrm{C}$ o R . Let's transform it so that the drawdown no longer appears as a result but as a component of the calculation.
By multiplying left and right by $\mathrm{R}^{-1}$ we have :

$$
\mathrm{T}=\mathrm{C} \circ \mathrm{R} \quad \Rightarrow \mathrm{~T} \circ \mathrm{R}^{-1}=\mathrm{C} \text { o } \mathrm{R} \text { o } \mathrm{R}^{-1}
$$

R is a threading, therefore a mapping, and in this case R o $\mathrm{R}^{-1}=\mathrm{I}$. So we have :

$$
\begin{array}{ll}
\mathrm{T}=\mathrm{C} \text { oR } & =>\mathrm{T} \text { o } \mathrm{R}^{-1}=\mathrm{C} \text { oI } \\
\mathrm{T}=\mathrm{C} \text { oR } \quad & =>\mathrm{T} \text { o } \mathrm{R}^{-1}=\mathrm{C}
\end{array}
$$

The formula $\mathrm{C}=\mathrm{T}$ o $\mathrm{R}^{-1}$ indicates that the peg-plan C is the drawdown that is obtained with as pegplan, the drawdown T , and as threading, $\mathrm{R}^{-1}$.


The drawdown $\mathrm{C}=\mathrm{To} \mathrm{R}^{-1}$ can also be written $\mathrm{C}=\mathrm{To} \mathrm{R}^{-1} \circ \mathrm{I}$, making T appear as treadling, R as tie-up and I as threading :


This implication means that for any cloth of the "peg-plan - threading" type, the peg-plan is the result of the calculation of the cloth having : the initial threading as tie-up and the initial drawdown as treadling.

The game seems won ; the drawdown is an element of the calculation, it commands the peg-plan. By drawing another graphic in this "drawdown treadling", why wouldn't we obtain the corresponding peg-plan, by calculating this drawdown which reverses the roles? This would amount to using implication, which allowed us to go from one diagram to another, in the other direction.

We had the implication:
$\mathrm{T}=\mathrm{C}$ o $\mathrm{R}=>\mathrm{T}_{\mathrm{ol}} \mathrm{R}^{-1}=\mathrm{C}$
Let's look at its converse :
Suppose we start from the drawdown calculation $\mathrm{C}^{\prime}=\mathrm{T}^{\prime}$ o $\mathrm{R}^{-1}$ o I where $\mathrm{T}^{\prime}$ is the "treadling" in which we have drawn a new graphic (a circle), where R is the tie-up and where C ' is the "drawdown" result of the calculation.


$$
\mathrm{C}^{\prime}=\mathrm{T}^{\prime} \circ \mathrm{R}^{-1} \circ \mathrm{I}
$$

Let us consider the new drawdown $\mathrm{T}^{\prime \prime}=\mathrm{C}^{\prime}$ o R obtained by calculating the classical diagram with R as threading and $\mathrm{C}^{\prime}$, the peg-plan deduced from $\mathrm{T}^{\prime}$ as peg-plan. Is the drawdown $\mathrm{T}^{\prime \prime}=\mathrm{C}^{\prime}$ o R equal to $\mathrm{T}^{\prime}$ ?

Multiply on each side of the equality $\quad \mathrm{T}^{\prime} \circ \mathrm{R}^{-1}=\mathrm{C}^{\prime}$ par R :

$$
\mathrm{T}^{\prime} \circ \mathrm{R}^{-1}=\mathrm{C}^{\prime} \quad=>\quad \mathrm{T}^{\prime} \circ \mathrm{R}^{-1} \circ \mathrm{R}=\mathrm{C}^{\prime} \circ \mathrm{R}=\mathrm{T}^{\prime \prime}
$$

We know that $\mathrm{R}^{-1}$ o R is equal to I in the case where R is injective. R being a threading, it is a mapping. $R$ is not in general injective.
The reciprocal implication is therefore in general false, the drawdown $\mathrm{C}^{\prime}$ o R , ie. $\mathrm{T}^{\prime}$ o $\mathrm{R}^{-1}$ o R is not in general equal to $\mathrm{T}^{\prime}$.


However, even if $R$ is not injective, we have shown that $R^{-1} o \mathrm{R}$ always contains the diagonal I. We can therefore write :

| I | $\subset\left(\mathrm{R}^{-1} \circ \mathrm{R}\right)$ | $=>$ | $\mathrm{T}^{\prime} \circ \mathrm{I}$ | $\subset$ | $\left.\mathrm{T}^{\prime} \circ \mathrm{o}^{-1} \circ \mathrm{R}\right)$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| I | $\subset\left(\mathrm{R}^{-1} \circ \mathrm{R}\right)$ | $=>$ | $\mathrm{T}^{\prime}$ | $\subset$ | $\left(\mathrm{T}^{\prime} \circ \mathrm{R}^{-1} \circ \mathrm{R}\right)$ |
| I | $\subset\left(\mathrm{R}^{-1} \circ \mathrm{R}\right)$ | $=>$ | $\mathrm{T}^{\prime}$ | $\subset$ | $\mathrm{C}^{\prime} \circ \mathrm{R}$ |
| I | $\subset\left(\mathrm{R}^{-1} \circ \mathrm{R}\right)$ | $\Rightarrow$ | $\mathrm{T}^{\prime}$ | $\subset$ | $\mathrm{T}^{\prime \prime}$ |

The new drawdown $\mathrm{T}^{\prime \prime}=\mathrm{C}^{\prime}$ o R will therefore always contain $\mathrm{T}^{\prime}$. The graphics that we had drawn in $\mathrm{T}^{\prime}$ (the circle) will therefore be fully present in the drawdown $\mathrm{T}^{\prime \prime}=\mathrm{C}^{\prime}$ o R , but other points, due to the repetitions of threading on the same shaft in the threading ( to the fact that R is not injective), will be added to it. We will say that the drawdown $\mathrm{T}^{\prime \prime}=\mathrm{C}^{\prime}$ o R was generated by $\mathrm{T}^{\prime}$, and we will call the peg-plan $\mathrm{C}^{\prime}$ calculated according to $\mathrm{T}^{\prime}$, the peg-plan generated by $\mathrm{T}^{\prime}$. If this abstract analysis seems complicated to you, rest assured that the calculation of a generated drawdown is very simple in practice; "Pointcarré" takes care of all the calculations !


$$
\mathrm{T}^{\prime} \subset \mathrm{T}^{\prime \prime}
$$

The generated drawdown $\mathrm{T}^{\prime \prime}$ contains the circle in red $\mathrm{T}^{\prime}$ plus risers generated by $\mathrm{T}^{\prime}$ on R threading

Pointcarré allows you to draw directly in the drawdown in the representation
$\mathrm{T}=\mathrm{Cor}$.
You can even drag and drop the circle onto the drawdown.
The generated peg-plan is then calculated automatically, then the drawdown T is recalculated from this new peg-plan. The updated drawdown will contain the circle plus the extra risers that will be lifted by the generated peg-plan.

This example clearly shows how threading repeats and symmetries affect generated drawdown.

The graphics drawn in the drawdown not being a priori compatible with the standard weave structures of the threading, we will proceed to the setting in weave structures after the calculation of the generated peg-plan.


The generated drawdown set in weave structures
The great interest of this technique is to automatically develop a wide variety of graphics on the same threading. The previous example shows how to take into account the symmetries present in the threading to obtain a generated drawdown close to the initial graphic. A bad positioning of the graphic can generate parasites which mask the initial drawing. On the other hand, by using curves of great versatility (for example a pseudo-straight threading, see Method of the initials), one can produce, practically without constraints, the most diverse graphics.
chapter 6
drawdown symmetrical with respect to the first diagonal

## 1- "WEAVED AS DRAWN IN". <br> "TREADLING-TIE-UP-THREADING-DRAWDOWN" REPRESENTATION

We have already looked at the special case of a drawdown where the reversed threading is taken as weave structure (first part, B, chapter 2, 9-, b) ). Recall that a "weaved as drawn in" diagram in the "peg-plan-threading" type representation contains the diagonal I and is symmetrical with respect to it.


Threading axial $\mathrm{R}^{-1}$ o R

Let us continue this study with diagrams of the type "treadling-tie-up-threading-drawdown".
If tie-up A contains I , i.e. if it's a straight tie-up plus "something", it's clear that the drawdown $\mathrm{R}^{-1} \mathrm{o} \mathrm{A}^{-1} \mathrm{o} \mathrm{R}$ will contain the threading axial.


The tie-up A contains I,
so the drawdown $T=\mathrm{R}^{-1} \mathrm{o} \mathrm{A}^{-1}$ o R contains the threading axial (in red)

Let us see under which condition on A the symmetry with respect to the first diagonal is preserved:
To say that the drawdown $\mathrm{T}=\mathrm{R}^{-1} \mathrm{o} \mathrm{A}^{-1} \mathrm{o} \mathrm{R}$ is symmetrical is to say that it is equal to its reciprocal T-1

$$
\begin{array}{lll}
\mathrm{T}=\mathrm{T}^{-1} & <=> & \mathrm{R}^{-1} \circ \mathrm{~A}^{-1} \circ \mathrm{R}=\left(\mathrm{R}^{-1} \circ \mathrm{~A}^{-1} \circ \mathrm{R}\right)^{-1} \\
\mathrm{~T}=\mathrm{T}^{-1} & <=> & \mathrm{R}^{-1} \mathrm{oA} \mathrm{~A}^{-1} \circ \mathrm{R}=\mathrm{R}^{-1} \circ\left(\mathrm{~A}^{-1}\right)^{-1} \mathrm{o}\left(\mathrm{R}^{-1}\right)^{-1} \\
\mathrm{~T}=\mathrm{T}^{-1} & <=> & \mathrm{R}^{-1} \mathrm{oA}^{-1} \circ \mathrm{R}=\mathrm{R}^{-1} \circ \mathrm{~A} \circ \mathrm{R}
\end{array}
$$

R is a mapping because it is a threading, so we can simplify by R on the right :

$$
\mathrm{T}=\mathrm{T}^{-1} \ll>\quad \mathrm{R}^{-1} \mathrm{o} \mathrm{~A}^{-1}=\mathrm{R}^{-1} \mathrm{oA}
$$

R is a mapping so $\mathrm{R}^{-1}$ is injective, so we can simplify by $\mathrm{R}^{-1}$ on the left : $\mathrm{T}=\mathrm{T}^{-1}<=>\quad \mathrm{A}^{-1}=\mathrm{A}$
$\mathrm{A}=\mathrm{A}^{-1}$ means that A is symmetric. we can therefore state :
A "weaved as drawn in" diagram is symmetric with respect to the first diagonal if and only if its tieup is symmetric with respect to the first diagonal.

$$
\mathrm{T}=\mathrm{R}^{-1} \mathrm{o} \mathrm{~A}^{-1} \mathrm{o} \mathrm{R} \text { is symmetric }<=>\mathrm{A} \text { is symmetric }
$$



If the tie-up is structured around diagonal I. the drawdown will organize around the threading axial.


Let's examine the particular case of a return tie-up :


As expected, the drawdown is symmetrical with respect to the first diagonal because -I is symmetrical. This diagram is equivalent to the following "peg-plan-straight tie-up-threading" type diagram :


On the same threading we therefore now have two examples of diagrams of the "peg-plan-tie-upthreading" type producing a symmetrical curve with respect to the first diagonal to the drawdown : the threading axial $\mathrm{R}^{-1}$ o R and the previous case $\left(\mathrm{R}^{-1} \mathrm{o}-\mathrm{I}\right)$ o R . Threading axial is therefore only a special case and does not by itself highlight all the potentialities of threading. We are now going to undertake the systematic study of all the treadlings likely to produce, with a given threading, a symmetrical curve with respect to the first diagonal to the drawdown.

## 2- CONDITION FOR A CLOTH TO BE SYMMETRIC WITH REGARD TO THE FIRST DIAGONAL

We are looking for all symmetrical clothes having a given threading R
T being of the form $\mathrm{T}=\mathrm{X}$ o R
From the above
for any symmetric relation $S$, of the height of $R$ we have $T=R^{-1} o S^{-1} o \mathrm{R}$ is symmetric $\mathrm{T}=\left(\mathrm{R}^{-1} \mathrm{o}^{-1}\right) \mathrm{o}$ R is symmetric so $\mathrm{X}=\mathrm{R}^{-1}$ o $\mathrm{S}^{-1}$ is a solution.

We seek to show that all the solutions are of this form ( $\mathrm{R}^{-1} \mathrm{o}^{-1}$ with S symmetric ), which will allow us to say that there are as many symmetric clothes having R for threading, as symmetries of the threading R height.

Assume a symmetric cloth T with peg-plan X and threading R
T is of the form $\mathrm{T}=\mathrm{X} \circ \mathrm{R}$

\[

\]

If we put $\mathrm{S}=\mathrm{R}$ o X
X is of the form $\mathrm{X}=\mathrm{R}^{-1} \mathrm{o} \mathrm{S}^{-1}$
Let us show that S is symmetric.

| T is symmetric | $=>$ | $\mathrm{X}=\mathrm{R}^{-1} \circ(\mathrm{R} \circ \mathrm{X})^{-1} \quad$ from above |
| ---: | :--- | :--- |
| T is symmetric | $=>$ | $\mathrm{X}=\mathrm{R}^{-1} \circ \mathrm{~S}^{-1}$ |
| T is symmetric | $=>$ | $\mathrm{RoX}=\mathrm{R} \circ \mathrm{R}^{-1} \circ \mathrm{~S}^{-1}$ |
| T is symmetric | $=>$ | $\mathrm{RoX}=\mathrm{I} o \mathrm{~S}^{-1} \quad \mathrm{R} \circ \mathrm{R}^{-1}=\mathrm{I}$ because R is a mapping |
| T is symmetric | $=>$ | $\mathrm{S}=\mathrm{S}^{-1}$ |
| T is symmetric | $=>$ | S is symmetric |

We can therefore say that :
all symmetric clothes with a given threading R , of the form $\mathrm{T}=\mathrm{X}$ o R can be put in the form $\mathrm{T}=\mathrm{R}^{-1} \mathrm{o} \mathrm{S}^{-1}$ o R with S a symmetric relation such that $\mathrm{S}=\mathrm{R}$ o X .

To find all the symmetrical clothes T with threading R , it is enough to look for all the symmetrical relations $S$ of the height of $R$; the cloth can be expressed as a "weaved as draw in" cloth in the form $\mathrm{T}=\mathrm{R}^{-1}$ o $\mathrm{S}^{-1} \mathrm{o} \mathrm{R}$


A symmetrical cloth $\mathrm{T}=\mathrm{X}$ o R


X in the form of :
$\mathrm{X}=\mathrm{R}^{-1}$ o $\mathrm{S}^{-1}$


The calculation of the symmetry $\mathrm{S}=\mathrm{R}$ o X


T in the form of :
$\mathrm{T}=\mathrm{R}^{-1}$ o $\mathrm{S}^{-1}$ o R

## 3- PRACTICAL CONSEQUENCES

This result is particularly interesting for the study of block clothes.
Let's take this 4-block cloth as an example.


A symmetric relation of width n is completely determined the triangle of points equal to or greater than the first diagonal. It is enough to make a symmetry with respect to the first diagonal to obtain the lower triangle.
The number of points of this triangle is the sum of the first $n$ numbers, i.e. $n(n+1) / 2$.
The number of possibilities that these points are either black or white is $2^{n(n+1) / 2}$
There are therefore $2^{n(n+1) / 2}$ symmetric relations of width $n$.
For our example the number of symmetric relations is $2^{4(4+1) / 2}=2^{10}=1024$
It's a lot!
We will limit ourselves to tie-ups which raise only one block and only one per treadle.
The tie-up is therefore a mapping, it is square, it is therefore a bijection. This bijection is symmetric, so it is an involution.

It is therefore sufficient to use all the involutions of 4 width to have all the clothes symmetrical with respect to the first diagonal, where one block and only one is activated at each pick.

There are 10 involutions of 4 width :



## PART TWO

## TRANSFORMATION BASES

With a rigorous mathematical model of the cloth, we are going to develop in this second part new tools that will allow us to progressively move from a free drawing to an interpretation in shaft weaving.

## A THEORETICAL STUDY

Although we are getting closer to the weaving technique itself, we will start with a more theoretical setting up of the tools. They will then be implemented on specific examples. In the following paragraphs we will transform the threading, either by changing the order of the shafts, or by threading the ends of several shafts on the same shaft or by distributing the ends of one shaft on several others. The common point of all these manipulations is that we will always act globally at the shaft level. The relative arrangement of the ends on the same shaft remains unchanged.
chapter 1
Transformations preserving the dimension of the diagram. Amalgamations.

Most often for technical reasons, in particular to avoid friction and to facilitate the separation of the ends in tight warps, one has to change the order of the shafts of a loom. One may also want to evenly distribute the binding shafts among the pattern shafts. This "mixing" of shafts masks the geometrical characteristics of the threading, so rather than working directly on amalgamated threadings, with good technical characteristics, we propose to first study the threadings graphically, and then to arrange the shafts as well as possible only once the cloth is ready. This extra work in now negligible thanks to computer tools. In short, it is a matter of shuffling the shafts of a threading as one would shuffle a card game.

## 1- REARRANGEMENT BASES

How to represent the action of rearranging the shafts of a threading using a calculation. At the beginning a threading $R$, at the end a threading $R^{\prime}$ which takes again the shafts of $R$, but in a different order.

$\mathrm{T}=\mathrm{Mo}^{-1} \mathrm{o} \mathrm{R}$

Let's take a particular case of rearrangement of the threading of this cloth : let's rearrange the threading R by arranging its shafts in the reverse order. From last to first. The threading R' obtained is the symmetric of R with respect to the horizontal. We saw in the first part that the symmetric/ horizontal of R is written - I o R . Here is our calculation!

$R^{\prime}=-I$ o R
-I is a relation which makes it possible to transform R by making the first shaft correspond to the last and so on. The diagram of -I is that of the second diagonal, here an eight square diagonal. In column 1 the square 8 is checked, in column 2 the square $8-1=7$ is checked. in column 3 the square 6 is checked, etc. -I is the "correspondence table" between the shafts of R and $\mathrm{R}^{\prime} .-\mathrm{I}$ is a bijection, to each shaft corresponds a shaft and only one.

To rearrange the shafts of R in another order, it suffices to use another bijection B associating the shafts in a different order.


We will call such a bijection B a rearrangement base ; in mathematics we will speak of permutation.

If we replace in the cloth the old threading $R$ by the new threading $\mathrm{R}^{\prime}$ or $\mathrm{R}_{\mathrm{b}}$, the cloth will of course be affected. Indeed the treadles, no longer controlling the same shafts, will no longer raise the same ends. To keep the initial cloth, just transform the tie-up by changing the order of its lines in the same way as in threading ; thus the treadles will control the same shafts again.
This is simply demonstrated:
if we put $\quad R_{b}=B$ o $R \quad$ the threading rearranged by $B$
$\mathrm{A}_{\mathrm{b}}=\mathrm{B}$ o A the tie-up rearranged by B


Compare the initial cloth $\mathrm{T}=\mathrm{Mo} \mathrm{A}^{-1} \mathrm{o} \mathrm{R}$ with the cloth $\mathrm{T}_{\mathrm{b}}=\mathrm{Mo} \mathrm{A}_{\mathrm{b}^{-1}} \mathrm{o} \mathrm{R}_{\mathrm{b}}$
$\mathrm{T}_{\mathrm{b}}=\mathrm{MoA}_{b}{ }^{-1}$ o $\mathrm{R}_{\mathrm{b}}$
$\left.\mathrm{T}_{\mathrm{b}}=\mathrm{Mo(BoA}\right)^{-1} \mathrm{o}$ (BoR)
$\mathrm{T}_{\mathrm{b}}=\mathrm{Mo}\left(\mathrm{A}^{-1} \circ \mathrm{O}^{-1}\right) \mathrm{o}(\mathrm{BoR})$
$\mathrm{T}_{\mathrm{b}}=\mathrm{MoA}^{-1} \mathrm{o}\left(\mathrm{B}^{-1}\right.$ o B) o R
$\mathrm{T}_{\mathrm{b}}=\mathrm{MoA}^{-1}$ oI o $\mathrm{R} \quad \mathrm{B}$ is a bijection so $\mathrm{B}^{-1}$ o $\mathrm{B}=\mathrm{I}$
$\mathrm{T}_{\mathrm{b}}=\mathrm{MoA}^{-1} \mathrm{o}$ R
$\mathrm{T}_{\mathrm{b}}=\mathrm{T} \quad$ The two clothes are therefore identical.


One can rearrange the shafts of a threading in any order, without changing the drawdown of a Treadling-Tie-up-Threading representation cloth, as long as the lines of the tie-up are rearranged simultaneously in the same order.

## 2- AMALGAMING

What is the purpose of amalgamation? It is to move away the heddles containing neighboring ends. One of the solutions is to rearrange the threading by taking the shafts in the same order, but by shifting them two by two on the even shafts and then on the odd shafts.

To obtain automatically the amalgamated threading $\mathrm{R}_{\mathrm{a}}$, let us compute the drawdown $\mathrm{B}_{\mathrm{a}}$ o R , by choosing for bijection $\mathrm{B}_{\mathrm{a}}$ this shift base.


$$
\mathrm{R}_{\mathrm{a}}=\mathrm{B}_{\mathrm{a}} \mathrm{oR}
$$

To obtain the corresponding amalgamated tie-up $\mathrm{A}_{\mathrm{a}}$ (whose lines have been rearranged in the same way), it is sufficient to calculate $\mathrm{B}_{\mathrm{a}}$ o A.


$$
\mathrm{A}_{\mathrm{a}}=\mathrm{B}_{\mathrm{a}} \mathrm{o} \mathrm{~A}
$$

We find the original drawdown, but here the neighboring heddles are separated by at least one shaft.

H

$$
\mathrm{T}_{\mathrm{a}}=\mathrm{Mo}_{\mathrm{a}}{ }^{-1} \mathrm{o} \mathrm{R}_{\mathrm{a}}=\mathrm{Mo}\left(\mathrm{~B}_{\mathrm{a}} \circ \mathrm{oA}\right)^{-1} \mathrm{o}\left(\mathrm{~B}_{\mathrm{a}} \circ \mathrm{O}\right)=\mathrm{T}
$$

Our knowledge of weaving will help us to find the optimum base for amalgamation.
It is a question of cyclically associating the 8 numbers, in a regular way, so that their difference is the greatest possible. It is a satin that we need ! Indeed, what is a satin, if not a weave structure where we try to distribute the bindings, in a regular way, so that they are as far as possible from each other. The largest possible shift for an 8 satin is 5 , so the best base for amalgamation is an 8 satin with a 5 shift.


$$
\mathrm{R}_{\mathrm{s}}=\mathrm{B}_{\mathrm{s}} \circ \mathrm{R}
$$



$$
\mathrm{A}_{\mathrm{s}}=\mathrm{B}_{\mathrm{s}} \mathrm{o} \mathrm{~A}
$$

Shifting cyclically by hand the 8 shafts of a threading from five to five requires attention. The cloth calculation function allows us to operate quickly and without error, on the threading, then on the tieup.

$\mathrm{T}_{\mathrm{s}}=\mathrm{Mo} \mathrm{As}_{\mathrm{s}}{ }^{-1} \mathrm{o} \mathrm{R}_{\mathrm{s}}=\mathrm{Mo}\left(\mathrm{B}_{\mathrm{s}} \circ \mathrm{A}\right)^{-1} \mathrm{o}\left(\mathrm{B}_{\mathrm{s}} \circ \mathrm{R}\right)=\mathrm{T}$
We now have a technically valid threading, but the graphics have been completely faded.

To obtain the new threading $\mathrm{R}_{\mathrm{s}}$ and the new tie-up $\mathrm{A}_{\mathrm{s}}$ amalgamated by the base $\mathrm{B}_{\mathrm{s}}$, we calculated two cloth diagrams : $\mathrm{R}_{\mathrm{s}}=\mathrm{B}_{\mathrm{s}}$ o R and $\mathrm{A}_{\mathrm{s}}=\mathrm{B}_{\mathrm{s}} \mathrm{o} \mathrm{A}$. In fact a single calculation would have been enough, using a multiple cloth diagram. By constructing a composite threading with R and A side by side, we could calculate a single drawdown, with $\mathbf{B}_{\mathrm{s}}$ as treadling; we would obtain in the drawdown part, side by side the two results $\mathrm{R}_{\mathrm{s}}$ and $\mathrm{A}_{\mathrm{s}}$.


In fact we will see that there is an even more clever way to do this.
In all these transformations we have not touched the treadling of the cloth. For a handwoven look this is important. The aim is to keep the number of treadles to a minimum and to make the treadling pattern easy to follow and remember. In the previous example the graphic aspect of the threading has completely disappeared, but the treadling has remained intact.
From an industrial perspective it is more natural to act on the peg-plan. That is why we will discuss another technique of amalgamating a cloth.

## 3- MULTIPLE AMALGAMATION DIAGRAM

Let's transform our cloth in the "peg-plan-threading" representation (see part 1, C, chapter 2).


How to act on the peg-plan C so that the amalgamated threading cloth remains unchanged ? Let's take the example of an amalgamation on the even and odd shafts.

In the representation with tie-up we saw that it was sufficient to rearrange the lines of threading R and tie-up A with the same bijection $\mathrm{B}_{\mathrm{a}}$.

The amalgamated threading is written $R_{a}=B_{a} o R$
The amalgamated tie-up is written $A_{a}=B_{a}$ o A
The amalgamated drawdown $T_{a}$ is equal to the starting drawdown $T$

In the representation without tie-up, the cloth is written
$\mathrm{T}=\mathrm{C}$ o R with the peg-plan $\mathrm{C}=\mathrm{Mo} \mathrm{A}^{-1}$
From the above we have $\mathrm{T}=\mathrm{Mo} \mathrm{A}^{-1} \mathrm{o}_{a^{-1}} \mathrm{o}_{\mathrm{a}} \mathrm{o}$ R
let $\mathrm{T}=\mathrm{Co} \mathrm{o}_{\mathrm{a}^{-1}}$ o $\mathrm{Ra}_{\mathrm{a}}$
If we note $\mathrm{C}_{\mathrm{a}}$ the peg-plan C transformed by the reciprocal of the bijection $\mathrm{B}_{\mathrm{a}}, \mathrm{C}_{\mathrm{a}}=\mathrm{C}$ o $\mathrm{Ba}_{a^{-1}}$
We can write $\mathrm{T}=\mathrm{C}_{\mathrm{a}} \mathrm{o} \mathrm{R}_{\mathrm{a}}$
In another way we can write :
$\mathrm{C}_{\mathrm{a}}$ o $\mathrm{R}_{\mathrm{a}}=\mathrm{C}$ o $\mathrm{Ba}_{\mathrm{a}}{ }^{-1}$ o $\mathrm{B}_{\mathrm{a}}$ o R
$\mathrm{C}_{\mathrm{a}}$ o $\mathrm{R}_{\mathrm{a}}=\mathrm{C}$ o I o $\mathrm{R} \quad \mathrm{B}_{a^{-1}}$ o $\mathrm{B}_{\mathrm{a}}=\mathrm{I}$, because $\mathrm{B}_{\mathrm{a}}$ is a bijection
$\mathrm{C}_{\mathrm{a}} \circ \mathrm{R}_{\mathrm{a}}=\mathrm{C} \circ \mathrm{R}$
$\mathrm{C}_{\mathrm{a}}$ o $\mathrm{R}_{\mathrm{a}}=\mathrm{T}$
The shafts of a threading can be rearranged in any order, without changing the drawdown of a Peg-Plan-Threading representation clothg, as long as the columns of the peg-plan are simultaneously rearranged in the reciprocal order.

More simply we can say that in order to keep the drawdown unchanged, we just need to rearrange the peg-plan columns so that each peg-plan column always corresponds to the same threading shaft.


$\mathrm{T}_{\mathrm{a}}=\mathrm{C}_{\mathrm{a}}$ o $\mathrm{R}_{\mathrm{a}}=\left(\mathrm{C}\right.$ o $\left.\mathrm{B}_{\mathrm{a}}{ }^{-1}\right)$ o $\left(\mathrm{B}_{\mathrm{a}}\right.$ o R$)=\mathrm{C}$ o I o $\mathrm{R}=\mathrm{C}$ o $\mathrm{R}=\mathrm{T}$

We will group all these calculations on a multiple cloth diagram. This way we will be able to switch automatically from the initial cloth diagram to the amalgamated cloth diagram with a single calculation.
To do this we will first present the calculation of the amalgamated threading $\mathrm{R}_{\mathrm{a}}$ and the amalgamated peg-plan $C_{a}$ in a different way, by operating the reciprocal of the base $B_{a}{ }^{-1}$, or the $B_{a}$ base in the tie-up (see part one, D, chapter 1).

$\mathrm{R}_{\mathrm{a}}=\mathrm{Io}\left(\mathrm{Ba}_{\left.\mathrm{a}^{-1}\right)^{-1} \mathrm{o}} \mathrm{R}\right.$
$\mathrm{R}_{\mathrm{a}}=\mathrm{B}_{\mathrm{a}} \mathrm{o} \mathrm{R}$

By reconciling the calculation of the new
threading $\mathrm{R}_{\mathrm{a}}=\mathrm{B}_{\mathrm{a}}$ o R
and that of the new peg-plan
$\mathrm{C}_{\mathrm{a}}=\mathrm{C}$ o $\mathrm{B}_{\mathrm{a}}{ }^{-1}$
until the two identities I are merged, we obtain the multiple diagram we are looking for.

Let us add I in the square of the top right corner and examine the four true calculations of this multiple diagram.


The two equivalent clothes on a single multiple diagram, $\mathrm{T}=\mathrm{C}$ o R and $\mathrm{T}=\mathrm{C}_{\mathrm{a}}$ o $\mathrm{R}_{\mathrm{a}}$

On the outside top right, the 4 framed diagrams show the actual calculation done by the computer. These diagrams are made of :

- a composite treadling including the peg-plan $\mathrm{C}_{\mathrm{a}}$ topped (orange line) by the base $\mathrm{B}_{\mathrm{a}}$
- a straight tie-up
- a composite threading including the $\mathrm{R}_{\mathrm{a}}$ threading extended (orange line) to the right of the $\mathrm{Ba}^{-1}$ reciprocal of the $\mathrm{B}_{\mathrm{a}}$ base.
- a cloth, on a grey background, which has the appearance (red lines) of a second set of 4 diagrams forming the amalgamated cloth.

We will see that the real calculations made by the computer and the apparent calculations are equivalent.

The 4 calculations made by the computer, in green :


The initial cloth
$\mathrm{T}=\mathrm{Co} \mathrm{o}^{-1}$ o R

$$
\mathrm{T}=\mathrm{C} \circ \mathrm{o}
$$



The amalgamated peg-plan
$\mathrm{C}_{\mathrm{a}}=\mathrm{Co} \mathrm{o} \mathrm{I}^{-1}$ o $\mathrm{Ba}_{a^{-1}} \quad \mathrm{C}_{\mathrm{a}}=\mathrm{Co} \mathrm{o}_{\mathrm{a}^{-1}}$


The amalgamated threading
$\mathrm{R}_{\mathrm{a}}=\mathrm{Ba}_{\mathrm{a}} \mathrm{ol}^{-1}$ or $\quad \mathrm{R}_{\mathrm{a}}=\mathrm{B}_{\mathrm{a}}$ o R


The straight tie-up
$\mathrm{B}_{\mathrm{a}}$ o $\mathrm{I}^{-1} \mathrm{oB}_{\mathrm{a}}{ }^{-1}=\mathrm{B}_{\mathrm{a}}$ o $\mathrm{Ba}^{-1}=\mathrm{I}$
$B_{a}$ o $B_{a^{-1}}=I \quad$ because $B_{a}$ is a bijection.

The 5 apparent cloth calculations that are all correct.


The amalgamated cloth
$\mathrm{T}_{\mathrm{a}}=\mathrm{C}_{\mathrm{a}}$ o $\mathrm{I}^{-1}$ o $\mathrm{R}_{\mathrm{a}}=\left(\mathrm{Co} \mathrm{o}_{\mathrm{a}}{ }^{-1}\right)$ o $\left(\mathrm{B}_{\mathrm{a}} \circ \mathrm{oR}\right)$
$\mathrm{T}_{\mathrm{a}}=\mathrm{CoIoR}$
$\mathrm{T}_{\mathrm{a}}=\mathrm{CoR}=\mathrm{T}$



$$
\begin{aligned}
& \mathrm{T}_{\mathrm{a}}=\mathrm{C}_{\mathrm{a}} \mathrm{O}\left(\mathrm{~B}_{\mathrm{a}}{ }^{-1}\right)^{-1} \mathrm{o} \mathrm{R} \\
& \mathrm{~T}_{\mathrm{a}}=\mathrm{C} \text { o } \mathrm{Ba}^{-1} \text { o }\left(\mathrm{B}_{\left.\mathrm{a}^{-1}\right)^{-1} \text { o } \mathrm{R}, ~}^{\text {a }}\right. \\
& \mathrm{T}_{\mathrm{a}}=\mathrm{C} \text { o I o } \mathrm{R}=\mathrm{Cor}=\mathrm{T}
\end{aligned}
$$


$\mathrm{T}=\mathrm{C}$ o $\mathrm{Ba}^{-1}$ o $\mathrm{R}_{\mathrm{a}}=\mathrm{C}$ o $\mathrm{B}_{\mathrm{a}}{ }^{-1}$ o $\mathrm{B}_{\mathrm{a}}$ o R $\mathrm{T}=\mathrm{C}$ o I o $\mathrm{R}=\mathrm{C}$ o R

$\mathrm{C}_{\mathrm{a}}=\mathrm{C}$ o $\mathrm{B}_{\mathrm{a}}{ }^{-1}$ o I
$\mathrm{C}_{\mathrm{a}}=\mathrm{C}$ o $\mathrm{B}_{\mathrm{a}^{-1}}$

Indeed, the multiple cloth diagram can only be read as the display of two equivalent clothes : the initial cloth and the amalgamated cloth.


The initial cloth
$\mathrm{T}=\mathrm{Co} \mathrm{o}^{-1}$ o R
$\mathrm{T}=\mathrm{Cor}$


The amalgamated cloth

$$
\begin{aligned}
& \mathrm{T}_{\mathrm{a}}=\mathrm{C}_{\mathrm{a}} \text { o } \mathrm{I}^{-1} \text { o } \mathrm{R}_{\mathrm{a}}=\mathrm{C}_{\mathrm{a}} \text { o } \mathrm{R}_{\mathrm{a}} \\
& \mathrm{~T}_{\mathrm{a}}=\left(\mathrm{Co}_{\mathrm{a}} \text { o } \mathrm{a}^{-1}\right) \text { o }\left(\mathrm{B}_{\mathrm{a}} \text { o } \mathrm{R}\right) \\
& \mathrm{T}_{\mathrm{a}}=\mathrm{C} \text { o I or } \\
& \mathrm{T}_{\mathrm{a}}=\mathrm{C} \text { o } \mathrm{R}=\mathrm{T}
\end{aligned}
$$

To go from the initial cloth to an equivalent cloth where we mix the rows of the shafts and the columns of the peg-plan, we simply choose a bijection and replace the bijection $\mathrm{B}_{\mathrm{a}}$ and its reciprocal $\mathrm{Ba}_{a^{-1}}$ in the multiple cloth diagram.

For example, let's use the bijection, satin of $8, \mathrm{~B}_{\mathrm{s}}=$ which amalgamates the threading even more smoothly. Note that $B_{s}=B_{s}{ }^{-1}, B_{s}$ so Bs is symmetric, it is an involution.


The simple change of base, gives the new amalgamated cloth

## 4- EOUIVALENT DIAGRAMS

Let's consider the process we have just completed with a little hindsight. We have transformed a threading R to obtain the amalgamated threading $\mathrm{R}^{\prime}$ in composing R followed by a bijection $\mathrm{B}: \mathrm{R}^{\prime}=$ B o R . We know how to correct the peg-plan so that the drawdown is preserved, i.e. that the threadings R and $\mathrm{R}^{\prime}$ produce the same drawdown. We can even say more : all the drawdowns that we can obtain with threading R , we will also be able to obtain them with threading $\mathrm{R}^{\prime}$, since for each drawdown we are always able to find the peg-plan that will preserve it with threading R'. Intuitively we feel that R and $\mathrm{R}^{\prime}$ are not very different ; in fact they have the same shafts, but in a different order. We will say that threading $R^{\prime}$ is equivalent to threading $R$. More generally we will say :

A relation $\mathrm{A}^{\prime}$ is "equivalent, up to the lines order" to a relation A , if and only if there exists a bijection B such that $\mathrm{A}^{\prime}=\mathrm{B}$ o A

If $\mathrm{A}^{\prime}$ is "equivalent, up to the lines order" to A , as the composite of A followed by the bijection B , then A is "equivalent, up to the lines order" to $\mathrm{A}^{\prime}$ as the composite of $\mathrm{A}^{\prime}$ followed by the bijection $\mathrm{B}^{-1}$.

Indeed
$\mathrm{A}^{\prime}=\mathrm{B}$ о $\mathrm{A} \quad=>\mathrm{B}^{-1} \mathrm{o}^{\prime}=\mathrm{B}^{-1}$ o B o A
$\mathrm{A}^{\prime}=\mathrm{B}$ o $\mathrm{A} \quad=>\mathrm{B}^{-1}$ o $\mathrm{A}^{\prime}=\mathrm{I}$ o $\mathrm{A} \quad \mathrm{B}^{-1}$ o $\mathrm{B}=\mathrm{I}$ because B is a bijection
$\mathrm{A}^{\prime}=\mathrm{B}$ o $\mathrm{A} \quad=>\mathrm{B}^{-1}$ o $\mathrm{A}^{\prime}=\mathrm{A}$
$\mathrm{A}^{\prime}=\mathrm{B}$ o $\mathrm{A} \quad=>\mathrm{A}=\mathrm{B}^{-1}$ o $\mathrm{A}^{\prime} \quad \mathrm{A}$ is "equivalent, up to the lines order" to $\mathrm{A}^{\prime}$ as the composite of $\mathrm{A}^{\prime}$ followed by the bijection $\mathrm{B}^{-1}$.

We will then say that the relations A and A' are "equivalent, up to the lines order".

A relation $\mathrm{A}^{\prime}$ is "equivalent, up to the columns order" to a relation A , if and only if there exists a bijection B such that $\mathrm{A}^{\prime}=\mathrm{A}$ о B

In the same way we would show that :

If A ' is "equivalent, up to the columns order" to A , as the composite of the bijection B followed by A , then A is "equivalent, up to the columns order" à $\mathrm{A}^{\prime}$, as the composite of the bijection $\mathrm{B}^{-1}$ followed b A.

We will then say that the relations A and $\mathrm{A}^{\prime}$ are "equivalent, up to the columns order".
We will simply speak of "equivalent threadings" for threadings, "equivalent, up to the lines order". We will simply speak of "equivalent peg-plans" for peg-plans, "equivalent, up to the columns order"

We will now transform the threadings with a similar technique, i.e., replacing a threading R with the threading $\mathrm{R}^{\prime}=\mathrm{B}$ o R , but choosing B so as to modify the threadings more deeply.
chapter 2
Transformations decreasing the dimension of the diagram. Telescoping.

The difficulty in interpreting a curve in a weaving diagram comes mainly from the fact that a threading has only a very limited number of shafts. How can you fit a curve drawn on a 48 lines layout sheet into a threading with 12 shafts? This is the problem we propose to solve in this chapter. There are two main families of solutions :

- reduce the vertical definition of the curve, which has the advantage of preserving the graphics of the surfaces, and the disadvantage of giving a staircase profile to the initial curve.
- cutting the curve into several slices that are superimposed, which has the advantage of preserving the entire profile of the initial curve, but has the disadvantage of creating parasitic curves that affect the perception of the initial graphics.

Let's start with the first family of solutions.


This cloth has a threading of 48 shafts. How do you reduce it to 12 ? Just divide by four !
Let's build from this threading R a new threading $\mathrm{R}^{\prime}$ : let's make in the threading R packages of four shafts and let's put all the ends of these four shafts on one of the threading R'.


The ends of 4 shafts on a green background, above, are grouped on a single shaft, below
As for the amalgamation, let us represent this transformation by a calculation :
Let's call B the relation that allows us to go from threading R to threading R'. The first package of four shafts of R, shafts $1,2,3$ and 4 will be associated with shaft 1 of R'. The next four shafts are associated with shaft 2 and so on. In the first four columns of $B$ a square will be checked on line 1 , in the next four columns a square will be checked on line 2 and so on. B is a stretched diagonal formed by small horizontal segments of four squares.


Each of the shafts of $R$ is associated to one and only one shaft of $R^{\prime}$ : the relation $B$ is a mapping. Different shafts of the threading $R$ are associated to the same shaft of $R^{\prime}$ : the relation $B$ is not injective.
Contrary to the rearrangement bases this relation is not a bijection; it is rectangular. B is wider than high and has the property of a threading : it is a mapping. The relation serves here to reduce the definition, i.e. to digitize the threading R :

We will call the mapping $B$ a digitizing base.

$R^{\prime}$ is deduced from $R$ by the formula $R^{\prime}=B$ o $R$. This calculation is presented by considering the reciprocal $B^{-1}$ of $B$ as a tie-up : $R^{\prime}=B$ o $R=I$ o $\left(B^{-1}\right)^{-1}$ o $R$

To amalgamate a cloth, after having amalgamated the threading, we had looked for how to transform the peg-plan in order to find again the initial cloth. Here, the threadings R and $\mathrm{R}^{\prime}$ are not equivalent because $\mathrm{R}^{\prime}$ is the product of R by a mapping and not by a bijection. $\mathrm{R}^{\prime}$ cannot produce all the clothes that R can produce. $\mathrm{R}^{\prime}$ cannot even be woven with the peg-plan C because $\mathrm{R}^{\prime}$ and R do not have the same number of shafts. Our goal here is to construct a new cloth, which we will call $\mathrm{T}^{\prime}$, that has four times fewer shafts than the original cloth $\mathrm{T}=\mathrm{C}$ o R . We have threading R ' on four times fewer shafts, let's reduce the number of treadles of peg-plan C in the same way, by dividing its width by four to get the new peg-plan $\mathrm{C}^{\prime}$.


We pass from pegplan C to peg-plan $\mathrm{C}^{\prime}$ by the formula :
$\mathrm{C}^{\prime}=\mathrm{C}$ o $\mathrm{B}^{-1}$ o I
$\mathrm{C}^{\prime}=\mathrm{C}$ о $\mathrm{B}^{-1}$

With the threading $\mathrm{R}^{\prime}$ and the peg-plan $\mathrm{C}^{\prime}$ we obtain the new digitized cloth $\mathrm{T}^{\prime}=\mathrm{C}^{\prime}$ o $\mathrm{R}^{\prime}$ :


Cloth $\mathrm{T}^{\prime}$ has 12 shafts and 12 treadles, its graphic line is the same as that of the initial cloth T, but its four times lower definition gives it a staircase profile.

You are now familiar with multiple diagrams, and, at this point, it is natural that you ask yourself this question : how do you automatically switch from the diagram of the initial cloth T to the digitized cloth T'?

Let's put together the two computational diagrams of the new threading $\mathrm{R}^{\prime}$ and the new peg-plan $\mathrm{C}^{\prime}$ by combining their square I :


How to choose the missing tie-up in the upper left corner so that the calculation of the drawdown on the outer squares C and R gives the drawdown $\mathrm{T}^{\prime}$ as result? If we call A this tie-up, the cloth is written : $\mathrm{T}^{\prime}=\mathrm{C}_{\text {o }} \mathrm{A}^{-1}$ o R

The cloth $\mathrm{T}^{\prime}$ is written :

$$
\begin{aligned}
& \mathrm{T}^{\prime}=\mathrm{C}^{\prime} \text { o } \mathrm{R}^{\prime} \\
& \mathrm{T}^{\prime}=\left(\mathrm{C} \text { o } \mathrm{B}^{-1}\right) \text { o }(\mathrm{B} \text { o } \mathrm{R}) \\
& \mathrm{T}^{\prime}=\mathrm{C} \text { o }\left(\mathrm{B}^{-1} \circ \mathrm{o}\right) \circ \mathrm{R} \\
& \mathrm{~T}^{\prime}=\mathrm{C} \text { o }\left(\mathrm{B}^{-1} \circ \mathrm{~B}\right)^{-1} \circ \mathrm{o} \\
& \text { If we put } \mathrm{A}=\mathrm{B}^{-1} \circ \mathrm{O}
\end{aligned}
$$

we have shown that $\mathrm{B}^{-1} \mathrm{o} \mathrm{B}$ is symmetric (and contains I ) we have well $\mathrm{T}^{\prime}=\mathrm{C}$ o $\mathrm{A}^{-1}$ o R
$B$ is a non-bijective mapping and $B^{-1} o B$ is in general different from $I . B^{-1} o B$ is the threading axial of B (see Part I, B, 9, b) ).
We will say more simply that $B$ is the axial of the base $B$.


We know that the axial B contains I and is symmetric with respect to it.
For this digitizing base, the diagonal I is surrounded by 4 X 4 blocks.


Now that we have completed our multiple diagram, let's see if it works properly.

$\mathrm{T}^{\prime}=\mathrm{C}^{\prime}$ o $\mathrm{R}^{\prime}$
This calculation of the cloth $\mathrm{T}^{\prime}$, is a possible reading in the multiple diagram, it is not made by the computer from $\mathrm{C}^{\prime}$ and de $\mathrm{R}^{\prime}$.


$$
\begin{aligned}
& T^{\prime \prime}=\mathrm{Co} \circ\left(\mathrm{~B}^{-1} \circ \mathrm{~B}^{-1} \circ \mathrm{R}\right. \\
& \mathrm{T}^{\prime \prime}=\mathrm{C} \circ \mathrm{~B}^{-1} \circ\left(\mathrm{~B}^{-1}\right)^{-1} \circ \mathrm{R} \\
& \mathrm{~T}^{\prime \prime}=\mathrm{Co} \circ \mathrm{~B}^{-1} \circ \mathrm{~B} \circ \mathrm{R} \\
& \mathrm{~T}^{\prime \prime}=\left(\mathrm{Cob} \mathrm{~B}^{-1}\right) \circ(\mathrm{B} \circ \mathrm{R}) \\
& \mathrm{T}^{\prime \prime}=\mathrm{C}^{\prime} \circ \mathrm{R}^{\prime} \\
& \mathrm{T}^{\prime \prime}=\mathrm{T}
\end{aligned}
$$

This calculation of $\mathrm{T}^{\prime \prime}$ is done by the computer, when the multiple diagram is represented by a single cloth (the green diagrams).

There are two equivalent ways to see the digitized cloth $\mathrm{T}^{\prime}$ :

- from the starting threading $R$, the starting peg-plan $C$ and the tie-up $\left(B^{-1} o B\right)$, - or from the digitized peg-plan $\mathrm{C}^{\prime}$ and the digitized threading $\mathrm{R}^{\prime}$.

It is the interest of the multiple diagram to show that the two calculations are equivalent.
The cloth $\mathrm{T}^{\prime}=\mathrm{C}^{\prime}$ o $\mathrm{R}^{\prime}$ is on 12 shafts, instead of 48 for $\mathrm{T}=\mathrm{C}$ o R .
Moreover $\mathrm{T}^{\prime} \subset \mathrm{T}, \mathrm{T}$ contains the graph of T , plus harmonics due to the grouping of some shafts into one.

Moreover, the following two calculations of $\mathrm{C}^{\prime}$ and two calculations of $\mathrm{R}^{\prime}$ are well equivalent:

$\mathrm{C}^{\prime}=\mathrm{C}_{\mathrm{o}}$ o $\mathrm{B}^{-1}$ o I
$\mathrm{C}^{\prime}=\mathrm{C}$ о $\mathrm{B}^{-1}$
This calculation of the peg-plan $\mathrm{C}^{\prime}$, is a possible reading in the multiple diagram, it is not done by the computer.

$\mathrm{C}^{\prime}=\mathrm{Co}\left(\mathrm{B}^{-1} \text { o B }\right)^{-1}$ o $\mathrm{B}^{-1}$
$\mathrm{C}^{\prime}=\mathrm{Coo} \mathrm{B}^{-1} \mathrm{o}\left(\mathrm{B}^{-1}\right)^{-1}$ o $\mathrm{B}^{-1}$
$\mathrm{C}^{\prime}=\mathrm{C}$ о $\mathrm{B}^{-1}$ о B о $\mathrm{B}^{-1}$
$\mathrm{C}^{\prime}=\mathrm{C}$ o $\mathrm{B}^{-1}$ o I car B is a mapping $\mathrm{C}^{\prime}=\mathrm{C}$ o $\mathrm{B}^{-1}$
This calculation of $\mathrm{C}^{\prime}$ is done by the computer.

$\mathrm{R}^{\prime}=\mathrm{I} o\left(\mathrm{~B}^{-1}\right)^{-1} \mathrm{o}$ R
$\mathrm{R}^{\prime}=\mathrm{I}$ o B o R
$\mathrm{R}^{\prime}=\mathrm{B} \circ \mathrm{R}$

This calculation of the threading $R^{\prime}$, is a possible reading in the multiple diagram, it is not done by the computer.

$R^{\prime}=\mathrm{B}$ o $\left(\mathrm{B}^{-1} \text { o } \mathrm{B}\right)^{-1}$ o R
$\mathrm{R}^{\prime}=\mathrm{B}$ o $\mathrm{B}^{-1} \mathrm{o}\left(\mathrm{B}^{-1}\right)^{-1}$ o R
$\mathrm{R}^{\prime}=\mathrm{B}$ o $\mathrm{B}^{-1}$ o B o R
$\mathrm{R}^{\prime}=\mathrm{I}$ o B o R because B is a mapping $\mathrm{R}^{\prime}=\mathrm{B} \circ \mathrm{R}$
This calculation of $\mathrm{R}^{\prime}$ is done by the computer.

We can read two other calculations attached to the cloth $\mathrm{T}^{\prime}$ :

$\mathrm{T}^{\prime}=\mathrm{C}^{\prime} \mathrm{o}\left(\mathrm{B}^{-1}\right)^{-1} \mathrm{o} \mathrm{R}$
$\mathrm{T}^{\prime}=\mathrm{C}^{\prime}$ o B o R
$\mathrm{T}^{\prime}=\mathrm{C}^{\prime}$ o $\mathrm{R}^{\prime}$
This calculation of the cloth $\mathrm{T}^{\prime}$, is a possible reading in the multiple diagram, it is not done by the computer.

$\mathrm{T}^{\prime}=\mathrm{C}$ o $\mathrm{B}^{-1}$ o $\mathrm{R}^{\prime}$
$\mathrm{T}^{\prime}=\mathrm{C}^{\prime}$ o $\mathrm{R}^{\prime}$
This calculation of the cloth $\mathrm{T}^{\prime}$, is a possible reading in the multiple diagram, it is not done by the computer.

In the upper right corner we find the elements that define the telescoping ; B , $\mathrm{B}^{-1}$ and $\mathrm{B}^{-1} \circ \mathrm{~B}$ :

$\mathrm{I}=\mathrm{B}$ o $\left(\mathrm{B}^{-1} \mathrm{o} B\right)^{-1}$ o $\mathrm{B}^{-1}$
$\mathrm{I}=\mathrm{B}$ o $\mathrm{B}^{-1} \mathrm{o}\left(\mathrm{B}^{-1}\right)^{-1}$ o $\mathrm{B}^{-1}$
$I=B$ o $B^{-1}$ o B o $B^{-1}$
$\mathrm{I}=\mathrm{I}$ o $\mathrm{I} \quad$ because B is a mapping $\mathrm{I}=\mathrm{I}$

This calculation of I is made by the computer; it is right in the multiple diagram.

However, to build the multiple diagram, we start by calculating the tie-up $\mathrm{B}^{-1}$ o B in a small annexed cloth.

Practically to pass automatically from the diagram of a cloth to its diagram digitized by the base B, we will proceed in the following way :

- calculation of the axial $\mathrm{B}^{-1}$ o B of the digitization base B which will be taken as tie-up
- construction of a composite treadling including the peg-plan C surmounted by base B
- construction of a composite threading including the threading $R$ extended to the right by the reciprocal $\mathrm{B}^{-1}$ of the base.
- calculation of the multiple diagram of digitization.

We will thus have, on the same diagram, the threading and the peg-plan of the initial cloth, and the digitized cloth.

Let's take another example from the second family of solutions mentioned at the beginning of this chapter.
What is the base for slicing a threading ? We know that a straight repeat reproduces a diagram ; to reproduce four slices of 12 shafts, it is sufficient to use a base formed by four straight repeats by 12 squares :


The new base B
Let's calculate the axial of this base $\mathrm{B}^{-1} \mathrm{o}$ B :


It is now enough to replace in the multiple cloth, the 3 elements on green background : $B$ at the top of the peg-plan, $\mathrm{B}^{-1}$ on the right of the threading and the tie-up by $\mathrm{B}^{-1}$ o B to obtain the multiple diagram of telescoping :


The result of the calculation gives us the telescoped threading $\mathrm{R}^{\prime}$, the telescoped peg-plan $\mathrm{C}^{\prime}$ on a salmon background, and the telescoped drawdown $\mathrm{T}^{\prime}$ on a purple background.

The shafts of R have been stacked one on top of the other in 12 packs of 4 , like the tubes of a telescope ; this is why we will call B a telescoping base.

Remark :
We constructed the multiple diagram as a single cloth, using an artifice to include B at the top of the peg-plan, $\mathrm{B}^{-1}$ to the right of the threading. This artifice allows any user with a software that computes a cloth with tie-up, to build multiple diagrams.

There is a version of the Pointcarré software, Mac OS Carbon, where $B, B^{-1}, R^{\prime}$ and $C^{\prime}$ are additional autonomous diagrams. It is then sufficient to define $\mathbf{B}$ so that all the calculations of the other diagrams are then automatically taken care of by Pointcarré.
This version was unfortunately never commercialized.

The tie-up as a composition of the base B followed by its reciprocal $\mathrm{B}^{-1}$, always contains the identity I. It follows that the starting drawdown $T=C$ o $B^{-1} o \mathrm{R}$, shown in red, is included in $\mathrm{T}^{\prime}$.


The telescoped cloth contains the entire curve of the initial cloth, in red, but parasitic curves have been added to it.
We will call these parasitic curves telescoping harmonics, in the sense that they resonate around the initial curve. The choice of the telescoping base allows the control of the spatial distribution of these harmonics, which can then underline or mask the initial curve.
The discussion of this choice is the subject of part B of this second part.

Note that each cloth T' is defined only by a different tie-up. All bases equivalent to B give the same tie-up $\mathrm{B}^{-1}$ o B. To find the harmonics we can work on the starting cloth, with only one tie-up, and a cloth calculating the tie-up $\mathrm{B}^{-1}$ o B .
Once the harmonics are in place, we can calculate the telescoped threading and peg-plan R' and $\mathrm{C}^{\prime}$.


## chapter 3

Transformations increasing the dimension of the diagram Combinations of diagrams.

Our discussion of clothes that are symmetrical with respect to the first diagonal has highlighted the importance of threading in the graphical characteristics of a cloth. The possibilities of a threading vary between two extremes : on the one hand simple threadings derived from a straight threading, on the other hand complex threadings built from a graphical curve.

- A straight threading is able to generate a very large variety of clothes, but the repeat is limited to the number of shafts, i.e. to very few things.
- A complex threading is capable of producing a large repeat pattern, but, although it is possible to vary the pattern for which the threading was constructed, all the clothes woven with this threading will have a family resemblance in their patterns.
In short, threading is caught in the following dilemma : simplicity and universality versus complexity and specificity. How, in these conditions, to produce a cloth with two very different graphics of large repeats? The starting idea is simple: let's start from two known clothes, each with a complex threading producing a specific graph, and let's try to build a new threading able to simulate indifferently one or the other of these two threadings.
Let us consider two clothes, of the "peg-plan-threading" type, $\mathrm{T}_{1}=\mathrm{C}_{1} \circ \mathrm{R}_{1}$ and $\mathrm{T}_{2}=\mathrm{C}_{2} \circ \mathrm{R}_{2}$. The $\mathrm{R}_{1}$ threading has four shafts and the $\mathrm{R}_{2}$ threading has six ; both have the same width.

$\mathrm{T}_{1}=\mathrm{C}_{1}$ o $\mathrm{R}_{1}$

$\mathrm{T}_{2}=\mathrm{C}_{2}$ o $\mathrm{R}_{2}$

What is a threading ... a set of shafts that each carry a set of ends. To build a threading R that can simulate both $R_{1}$ and $R_{2}$, is to look for a set of shafts that can produce all possible arrangements of ends with $\mathrm{R}_{1}$ and with $\mathrm{R}_{2}$.

Construisons l'ensemble de shafts suivant :
A shaft that contains the ends threaded in both shaft 1 of $R_{1}$ and shaft 1 of $\mathbf{R}_{2}$.
A shaft that contains the threaded ends in both shaft 1 of $R_{1}$ and shaft 2 of $\mathbf{R}_{2}$.
A shaft that contains the threaded ends in both shaft 1 of $R_{1}$ and shaft 3 of $R_{2}$.
A shaft that contains the threaded ends in both shaft 1 of $R_{1}$ and shaft 4 of $\mathbf{R}_{2}$.
A shaft that contains the threaded ends in both shaft 1 of $R_{1}$ and shaft 5 of $R_{2}$.
A shaft that contains the threaded ends in both shaft 1 of $R_{1}$ and shaft 6 of $\mathbf{R}_{2}$.
Let us consider the set of ends of the warp. $R_{2}$ and $R_{1}$ being threadings of the same width, each end of the warp is threaded in one and only one shaft of $R_{1}$ and in one and only one shaft of $R_{2}$. In particular each end threaded in shaft 1 of $R_{1}$ is threaded in one of the shafts of $R_{2}$. As we have gone through all the shafts of $R_{2}$, we can say that each end of shaft 1 of $R_{1}$ is threaded in one of the shafts of the set we have just constructed. If we lift all these shafts at the same time, the ends lifted will be the same as those that would have been lifted by shaft 1 of $\mathrm{R}_{1}$ alone ; this set of shafts is able to simulate the action of shaft 1 of $\mathrm{R}_{1}$.
If we build a second set of shafts on the same model, but with shaft 2 of $R_{1}$, this set will be able to simulate shaft 2 of $R_{1}$. If we build as many sets of shafts of this type as there are shafts in $R_{1}$, we will be able to simulate the action of all the shafts of $\mathrm{R}_{1}$, i.e. to simulate the whole $\mathrm{R}_{1}$ threading. Indeed the set of all the shafts of all these sets of shafts forms a threading : each end of the warp being threaded in one and only one shaft of $\mathrm{R}_{1}$ will be threaded in one and only one of the sets of shafts that we have constructed ; moreover in each of these sets of shafts, each end is threaded in only one of the shafts because it is threaded in one and only one shaft of $\mathbf{R}_{2}$.
What would we have to do to be able to simulate $\mathrm{R}_{2}$ threading? The same thing, but reversing the roles of $\mathrm{R}_{1}$ and $\mathrm{R}_{2}$ ! Clearly, this new set of shafts would have exactly the same shafts, but arranged in a different order.
For example, to simulate the action of shaft 1 of $R_{2}$ it would be sufficient to lift the following shafts together :

A shaft that contains the threaded ends in both shaft 1 of $R_{1}$ and shaft 1 of $R_{2}$. A shaft that contains the threaded ends in both shaft 2 of $R_{1}$ and shaft 1 of $R_{2}$. A shaft that contains the threaded ends in both shaft 3 of $R_{1}$ and shaft 1 of $R_{2}$. A shaft that contains the threaded ends in both shaft 4 of $R_{1}$ and shaft 1 of $R_{2}$.

Each of these shafts is the same as the first shaft of each of the sets of shafts we had built to simulate $\mathrm{R}_{1}$.

All these shafts are made up of ends threaded on both one of the shafts of $R_{1}$, and on one of the shafts of $\mathrm{R}_{2}$. Graphically, to say that these ends belong to both of these shafts is to say that they belong to the intersection of these shafts.
The threading R , able to simulate both the threading $\mathrm{R}_{1}$ and the threading $\mathrm{R}_{2}$, will have as shafts all possible intersections between a shaft of $\mathrm{R}_{1}$ and a shaft of $\mathrm{R}_{2}$. The number of possible intersections is equal to the number of shafts of $R_{1}$ multiplied by the number of shafts of $R_{2}$.

In our example R will have 24 shafts, formed by the following intersections :

| Shafts of $\mathrm{R}_{1}$ | Shafts of $\mathrm{R}_{2}$ |
| :---: | :---: |
| shaft 1 | shaft 1 |
| shaft 1 | shaft 2 |
| shaft 1 | shaft 3 |
| shaft 1 | shaft 4 |
| shaft 1 | shaft 5 |
| shaft 1 | shaft 6 |
| shaft 2 | shaft 1 |
| shaft 2 | shaft 2 |
| shaft 2 | shaft 3 |
| shaft 2 | shaft 4 |
| shaft 2 | shaft 5 |
| shaft 2 | shaft 6 |
| shaft 3 | shaft 1 |
| shaft 3 | shaft 2 |
| shaft 3 | shaft 3 |
| shaft 3 | shaft 4 |
| shaft 3 | shaft 5 |
| shaft 3 | shaft 6 |
| shaft 4 | shaft 1 |
| shaft 4 | shaft 2 |
| shaft 4 | shaft 3 |
| shaft 4 | shaft 4 |
| shaft 4 | shaft 5 |
| shaft 4 | shaft 6 |

Threading $\mathrm{R}_{1} \quad$ Threading $\mathrm{R}^{\prime}{ }_{2}$

Rather than making all these intersections of shafts one by one and then grouping them together to form threading $R$, we will build two threadings $R^{\prime}{ }_{1}$ and $R^{\prime}{ }_{2}$ from $R_{1}$ and $R_{2}$, grouping all the shafts of $R_{1}$ on one side and all the shafts of $\mathbf{R}_{2}$ on the other side following the above layout ; then we will globally make the intersection of the diagrams of $\mathrm{R}^{\prime}{ }_{1}$, and $\mathrm{R}^{\prime}{ }_{2}$ to obtain threading R . To respect the habit of numbering the shafts from bottom to top, we will actually stack the shafts in reverse order. As usual, we will use two new bases $B_{1}$ and $B_{2}$ to go from $R_{1}$ to $R^{\prime}{ }_{1}$ and from $R_{2}$ to $R^{\prime}{ }_{2}$ using an automatic cloth diagram calculation :


The $B_{1}$ base could be called a stretching base, and the $R_{2}$ base a repeating base. The common point of these bases is that they are injective ; they have one cross and only one per line. Indeed each shaft of $\mathbf{R}_{1}\left(\right.$ or $\left.R_{2}^{\prime}\right)$ is the copy of a single shaft of $\mathbf{R}_{1}$ (or $\mathbf{R}_{2}$ ).
The transformation of $\mathrm{R}_{1}$ (or $\mathrm{R}_{2}$ ) into $\mathrm{R}^{\prime}$ (or $\mathrm{R}^{\prime}$ ) is expressed by the formula $\mathrm{R}^{\prime}{ }_{1}=\mathrm{B}_{1}$ o $\mathrm{R}_{1}$, (or $\mathrm{R}_{2}{ }_{2}=\mathrm{B}_{2}$ o $\mathrm{R}_{2}$ ); it increases the number of shafts of $\mathrm{R}_{1}\left(\right.$ or $\left.\mathrm{R}_{2}\right)$.

To keep the drawdown $\mathrm{T}_{1}$ and $\mathrm{T}_{2}$, let's calculate the peg-plans $\mathrm{C}^{\prime \prime}=\mathrm{C}_{1}$ o $\mathrm{B}_{1}{ }^{-1}$ and $\mathrm{C}^{\prime}{ }_{2}=\mathrm{C}_{2}$ o $\mathrm{B}_{2^{-1}}$.




The calculations of this multiple diagram must be read by adding to all the letters, an index 1 or 2 , according to whether one considers the cloth 1 or the cloth 2 .
Let us examine the four calculations made by the computer :

- The calculation of the cloth $\mathrm{T}=\mathrm{CoI}^{-1}$ o $\mathrm{R}=\mathrm{C}$ o R
- The product B o $\mathrm{B}^{-1}$
- The calculation of the peg-plan $\mathrm{C}^{\prime}=\mathrm{C} \mathrm{o} \mathrm{I}^{-1}$ o $\mathrm{B}^{-1}={\mathrm{C} \text { o } \mathrm{B}^{-1}}^{1}$
- The calculation of the threading $\mathrm{R}^{\prime}=\mathrm{B}$ o $\mathrm{I}^{-1}$ o $\mathrm{R}=\mathrm{B}$ o R
- In the result part of the calculation diagram, we find at the bottom left the drawdown.

The calculation made by the computer is :
$\mathrm{T}=\mathrm{Cool}^{-1}$ o R
$\mathrm{T}=\mathrm{Cor}$
We can read a second calculation of this drawdown, is well equivalent:
$\mathrm{T}^{\prime}=\mathrm{C}^{\prime}$ o $\left(\mathrm{B}\right.$ o $\left.\mathrm{B}^{-1}\right)$ o $\mathrm{R}^{\prime}$
$\mathrm{T}^{\prime}=\left(\mathrm{C} \circ \mathrm{B}^{-1}\right) \circ\left(\mathrm{B} \circ \mathrm{B}^{-1}\right) \circ(\mathrm{B} \circ \mathrm{R})$
$T^{\prime}=C$ o $\left(B^{-1} \circ B\right)$ o $\left(B^{-1} \circ B\right) \circ R$
$\mathrm{T}^{\prime}=\mathrm{C}$ o I o I o $\mathrm{R} \quad \mathrm{B}^{-1}$ o $\mathrm{B}=\mathrm{I} \quad$ because the base B is injective
$\mathrm{T}^{\prime}=\mathrm{C}$ o R
$\mathrm{T}^{\prime}=\mathrm{T}$

- The calculation made by the computer is: $\mathrm{Bo}^{-1} \mathrm{o}^{-1}=\mathrm{B}$ o $\mathrm{B}^{-1}$
- The calculation of the peg-plan $\mathrm{C}^{\prime}$ made by the computer is :
$\mathrm{C}^{\prime}=\mathrm{C}_{\mathrm{o}} \mathrm{I}^{-1}$ o $\mathrm{B}^{-1}$
$\mathrm{C}^{\prime}=\mathrm{C}$ o $\mathrm{B}^{-1}$
We can read a second calculation of the peg-plan $\mathrm{C}^{\prime}$ which is well equivalent :
$\mathrm{C}^{\prime}=\mathrm{C}$ o $\mathrm{B}^{-1}$ o $\left(\mathrm{B} \circ \mathrm{B}^{-1}\right)$
$\mathrm{C}^{\prime}=\mathrm{Coo}\left(\mathrm{B}^{-1} \circ \mathrm{ob}\right)$ o $\mathrm{B}^{-1}$
$\mathrm{C}^{\prime}=\mathrm{C}$ o I o $\mathrm{B}^{-1} \quad \mathrm{~B}^{-1}$ o $\mathrm{B}=\mathrm{I} \quad$ because the base B is injective
$\mathrm{C}^{\prime}=\mathrm{C}_{\mathrm{o}} \mathrm{B}^{-1}$
- The calculation of the threading $\mathrm{R}^{\prime}$ done by the computer is : $\mathrm{R}^{\prime}=\mathrm{B}$ o R

We can read a second calculation of the threading $\mathrm{R}^{\prime}$ which is well equivalent :
$R^{\prime}=\left(B \circ B^{-1}\right) o\left(B^{-1}\right)^{-1}$ o $R$
$\mathrm{R}^{\prime}=\mathrm{B}$ o $\mathrm{B}^{-1}$ o B o R
$\mathrm{R}^{\prime}=\mathrm{B}$ oI o $\mathrm{R} \quad \mathrm{B}^{-1}$ o $\mathrm{B}=\mathrm{I} \quad$ because the base B is injective
$\mathrm{R}^{\prime}=\mathrm{B}$ o R
Let us now proceed to the intersection of the threadings $\mathrm{R}^{\prime}{ }_{1} \cap \mathrm{R}^{\prime}$, to obtain the threading $\mathrm{R}^{\prime}$ :


$\mathrm{R}^{\prime}$ in grey, $\mathrm{R}_{2}{ }_{2}$ in pink, $\mathrm{R}_{1} \cap \mathrm{R}_{2}^{\prime}$ in dark red

$\mathrm{R}^{\prime}=\mathrm{R}^{\prime}{ }_{1} \cap \mathrm{R}^{\prime}{ }_{2}$ in dark red

Threading
$\mathrm{R}^{\prime}=\mathrm{R}^{\prime}{ }_{1} \cap \mathrm{R}^{\prime}{ }_{2}$
can produce either $\mathrm{T}_{1}$ or $\mathrm{T}_{2}$ cloth depending on whether one follows the peg-plan $\mathrm{C}_{1}{ }_{1}$ or the peg-plan $\mathrm{C}_{2}^{\prime}$
$\mathrm{R}^{\prime}=\mathrm{R}^{\prime}{ }_{1} \cap \mathrm{R}^{\prime}{ }_{2}$
$\mathrm{T}_{1}=\mathrm{C}_{1}^{\prime}$ o $\mathrm{R}^{\prime}$
$\mathrm{T}_{2}=\mathrm{C}_{2}^{\prime}$ or
$\mathrm{R}_{1}$ and $\mathrm{R}_{2}$ being threadings,
$\mathrm{R}^{\prime}=\mathrm{R}^{\prime}{ }_{1} \cap \mathrm{R}^{\prime}{ }_{2}$ is also one. Each end is threaded in one and only one shaft of $\mathrm{R}_{1}$ i.e. $c_{1}$, and in one and only one shaft of $\mathrm{R}_{2}, \mathrm{c}_{2}$. It therefore belongs to the single shaft of R' that combines c1 and c2 and only to this shaft.

On the other hand it is possible that some shafts of $\mathrm{R}^{\prime}$ are empty. This is the case in this example where R' has 12 empty shafts. It is enough to delete them, as well as the corresponding columns of the pegplan to obtain a threading which produces indifferently the clothes $\mathrm{T}_{1}$ or $\mathrm{T}_{2}$, on 12 shafts.


The purpose of this example was to show the power of the cloth formula model and multiple diagrams.
To directly find a minimum threading that produces either $\mathrm{T}_{1}$ or $\mathrm{T}_{2}$ clothes, all you have to do with Pointcarré is to juxtapose $\mathrm{T}_{1}$ and $\mathrm{T}_{2}$ in height, then analyze this new diagram with the "New analyzed weave" function.

Although it looks messy, the R' threading is able to generate two very structured families of graphics. This example is a good demonstration of the need for an abstract approach to be able to arrive at threadings with such graphic possibilities ; it is one of the great charms of shaft weaving that it highlights the structure underlying any geometric type of graphics.

The price to pay for combining two threadings, the number of shafts of the first threading multiplied by the number of shafts of the second, may seem heavy. Fortunately it can be lower in many cases. Imagine that we combine two threadings that respect the tabby, even ends odd ends, i.e. that are drawn on a "tabby network", or, if you prefer, on an straight 2 initial. For all even shafts, only the even squares will be checkable, and, for all odd shafts, only the odd squares will be checkable. An even shaft and an odd shaft will never have any ends in common, their intersection will be empty. In all possible intersections between the two threadings, about half will be between even and odd shafts, so half of the shafts in R' are empty. An empty shaft is a shaft that is useless! So we can remove it... The combination of a threading of $n$ shafts and a threading on $p$ shafts drawn on a "tabby network" gives a combined threading of $n p / 2$ shafts (or of $(n p+1) / 2$ if $n$ and p are both odd), and not of $n p$ as one might have supposed.
With threadings on networks of larger initials the number of shafts of the combined threading is dramatically smaller than the product of the number of shafts of the threadings. If the threadings have other common properties this number can be further reduced.
In fact the number of shafts of a combined threading is a function of the degree of similarity between the two starting threadings. In the end, if we combine two equivalent threadings, the combined threading will have the same number of shafts as the initial threadings ; in fact, it will be equivalent to them.

The combination of threadings offers many possibilities of research. Why not combine more than two threadings ? Combine conflicting graphic lines. Etc ...


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